Avanti Learning Centres
2016 Edition

9P01. Motion

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9P01.1 Distance, Displacement & Speed

CONCEPTS COVERED

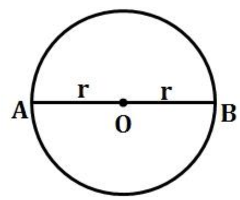
1. What is 'motion'?
2. Why is frame of reference important?
3. Relative motion and its significance in real life.
4. What is the difference between distance and displacement?
5. Uniform and non-uniform motion.
6. What is speed and how it is calculated?

IN CLASS EXERCISE

Q1. How can we describe the location of an object?

Q2. Differentiate between distance and displacement.

Q3. A particle is moving in a circular path of radius r. What will be its displacement after half a circle is completed?



Q4. A particle travels three quarters of a circle of radius r. What is the magnitude of its displacement?

Q5. A farmer moves along the boundary of a square field of side 10 m in 40 s. What will be the magnitude of displacement of the farmer at the end of 2 min 20 s from his initial position?

Q6. What is meant by uniform motion? Give an example.

Q7. Define the term average speed.

Q8. A car travels a certain distance with a speed of 50 km/hr and returns with a speed of 40 km/h. Calculate the average speed for the whole journey.

Q9. On a 100 km track, a train travels the first 30 km at a uniform speed of 30 $km\ /h$. How fast must the train travel the next 70 km so as to average 40 km/h for the entire trip?

HOMEWORK

Q1. With the help of a suitable example, explain the terms distance and displacement.

Q2. An object is moving with a uniform speed in a circle of radius r. Calculate the distance and displacement:

I. When it completes half of the circle.
II. When it completes a full circle.

Q3. A body moves in a circle of radius '$2R$'. What is the distance covered and displacement of the body after 2 complete rounds?

Q4. An athlete completes one round of a circular track of diameter 200 m in 40 s. What will be the distance covered and the displacement at the end of 2 min 20 s?

Q5. An ant climbs up five stairs, each of width 20 cm and height 20 cm. Find the distance covered and displacement of ant, if it starts from the bottom, and reaches the start of the sixth staircase.

Q6. What is non-uniform motion? Give some examples.

Q7. Does the speedometer of a car measure its average speed?

Q8. Abdul, while driving to school, computes the average speed for his trip to be 20 km/hr. On his return trip along the same route, there is less traffic and the average speed is 40 km/hr. What is the average speed for Abdul's entire trip?

Q9. A boy runs for 10 minutes at a uniform speed of 9 km/h. At what speed should he run for the next 20 minutes so that the average speed becomes 12 km/h?

ADVANCED QUESTIONS

Q1. A bridge is 500 m long. A 100 m long train crosses the bridge at a speed of 20 m/s. The time taken by train to cross it will be:

A) 25 s B) 5 s C) 30 s D) 20 s

Q2. A body covers one-third of its journey with speed u, next third with speed v and the last third with speed w. Calculate the average speed of the body over the entire journey.

Q3. When two bodies move uniformly towards each other, the distance between them decreases by 8 m/s. If both the bodies move in the same direction with the same speeds, the distance between them increases by 4 m/s. What are the speeds of the two bodies?

Q4. A man can row a boat at 1.8 $kmph$ in still water. He heads straight downstream for 2.7km in a river where the current is 0.9 $kmph$ and then returns to the starting point. Calculate the time for the round trip.

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9P01.2 Velocity & Acceleration

CONCEPTS COVERED

1. What is velocity?
2. How to find velocity from displacement?
3. What is the difference between average and instantaneous velocity?
4. What is acceleration?
5. What is positive and negative acceleration?
6. What do you mean by uniform and non-uniform acceleration?

IN CLASS EXERCISE

Q1. Define the term velocity. What is its SI unit? Is it a scalar or vector quantity?

Q2. Joseph jogs from one end A to the other end B of a straight 300 m road in 2 min 30 s and then turns around and jogs 100 m back to point C in another 50 s. What are Joseph's average speeds and velocities in jogging

I. From A to B II. From A to C?

Q3. What is acceleration? How will you express it mathematically?

Q4. When will you say a body is in (I) Uniform acceleration? (II) Non-uniform acceleration?

Q5. A bus retards uniformly at a rate of 3 m/s^2 and stops in 10 s. With what velocity was the bus travelling?

Q6. In your everyday life you come across a range of motions in which

I. Acceleration is in the direction of motion.
II. Acceleration is against the direction of motion.
III. Acceleration is uniform.
IV. Acceleration is non-uniform.

Can you identify one example each the above type of motion?

Q7. A body moving with an initial velocity of 5 m/s accelerates at 2 m/s^2. Its velocity after 10 s is

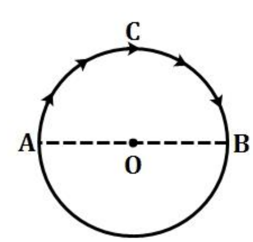
A) 20 m/s B) 25 m/s C) 5 m/s D) 22.5 m/s

Q8. Give examples where

I. The velocity is in opposite direction to the acceleration.
II. The velocity of the particle is zero but its acceleration is not zero.
III. The velocity is perpendicular to the acceleration.

HOMEWORK

Q1. An insect moves along a circular path of radius 10 cm with a constant speed. It takes 1 min to move from a point on the path to the diametrically opposite point. Find



I. The distance covered.
II. The speed.
III. The displacement.
IV. The average velocity.

Q2. I. An object travels 30 m in 4 s and then another 30 m is 2 s. What is the average speed of the object?
II. Is the data given above sufficient to find average velocity of the object?
III. Under what condition(s) is the magnitude of average velocity of an object equal to its average speed?

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Q3. State a relationship connecting u, v, a and t for an accelerated motion. Give an example of motion in which acceleration is uniform.

Q4. What is meant by retardation or deceleration?

Q5. The initial velocity of a train which is stopped in 20 s by applying brakes (retardation due to brakes being 1.5 m/s^2) is

A) 30 m/s B) 30 cm/s C) 20 cm/s D) 24 m/s

Q6. A car increases its speed from 20 km/h to 50 km/h in 10 seconds. Its acceleration is

A) 30 m/s^2 B) 3 m/s^2 C) 18 m/s^2 D) 0.83 m/s^2

Q7. A racer starts a car race from rest and finishes it with a velocity of 54 km/h. Assuming constant acceleration of 3 m/s^2, Find the time it will take to finish the race.

Q8. State the condition (s) under which the displacement of a particle and its average velocity are zero. What can be said about the corresponding average speed(s)?

Q9. An object does not possess an acceleration or retardation when it moves

A) In upward direction with decreasing speed
B) In downward direction with increasing speed
C) With constant speed along circular path
D) With constant speed along a particular horizontal direction

ADVANCED QUESTIONS

Q1. Can velocity of a particle be equal to zero at a given instant, and yet be varying with time? Explain with an example.

Q2. Two cars start off to race with velocities 4 m/s and 2 m/s and travel in a straight line with uniform accelerations 1 m/s^2 and 2 m/s^2 respectively. If they reach the final point at the same time instant, then what is the length of the path?
(Hint: When a body is travelling with uniform acceleration, distance travelled is given by the average velocity multiplied with the time of travel and the average velocity is the average of final and initial velocities)
(An equation in which the degree i.e. highest power present is two is known as a quadratic equation.
$x^2 - 6x = 0$ is an example of quadratic equation. It can be solved in the following manner:
$x(x - 6) = 0$
$x = 0$ or $x = 6$
These values are known as the roots of the above quadratic equation).

Q3. A 50g ball travelling at 25 m/s bounces off a wall and rebounds at 24 m/s. A high speed camera records this event. If the ball is in contact with the wall for 3.50 ms, what is the magnitude of the average acceleration of the ball during this time interval?

Q4. During a constant accelerated motion of a particle, average velocity of the particle

A) is always less than its final velocity
B) is always greater than its final velocity
C) may be zero
D) is half of its final velocity

Q5. A police jeep is chasing a thief with a velocity of 72 km/h. The thief is in another jeep with a velocity of 90 km/h. The police fires a bullet with a velocity of 180 km/h (with respect to the jeep) when the thief is 350 metres away. Determine the time after which the bullet will hit the thief.

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9P01.3 Graphical Representation of Motion & Uniform Circular Motion

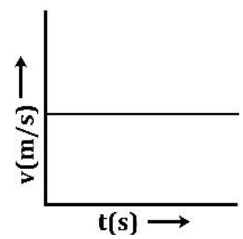
CONCEPTS COVERED

1. Introduction of graph plotting – origin, axes, coordinates
2. How to plot motion on a graph?
3. How to find speed/velocity from a distance/displacement time graph?
4. Calculating the distance travelled using the area under the graph.
5. How to find acceleration from velocity time graph?
6. Uniform circular motion and its various parameters (speed, direction) with examples.

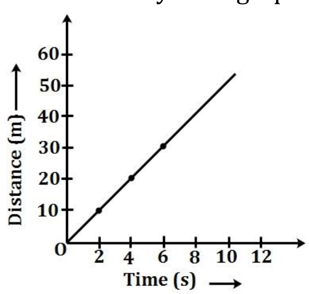
IN CLASS EXERCISE

Q1. What is the nature of distance time graphs for uniform and non-uniform motion of an object?

Q2. From the v-t graph alongside, (see figure), what can be inferred?

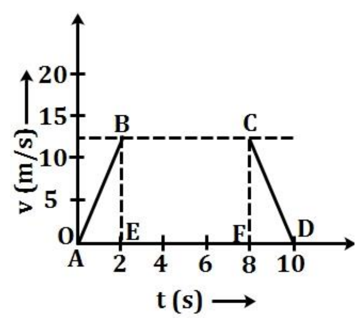


Q3. Make a velocity-time graph from the following displacement-time graph.



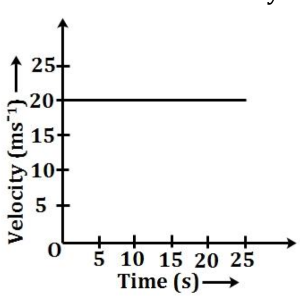
Q4. How do we measure magnitude of displacement from a v-t curve?

Q5. Find the displacement of a body whose velocity-time graph is shown alongside.



Q6. The velocity-time graph (see figure below) shows the motion of a cyclist. Find:

I. Its acceleration
II. Its velocity after 15 s and
III. The distance covered by the cyclist in 15 s.



Q7. A motorcyclist riding motorcycle A who is travelling at 36 km/h applies the brakes and stops the motorcycle in 10 s. Another motorcyclist of motorcycle B who is travelling at 18 km/h applies the brakes and stops the motorcycle in 20 s. Plot speed-time graph for the two motorcycles. Which of the two motorcycles travelled farther before it came to a stop?

Q8. Can an object be accelerated if it is moving with constant speed? Justify your answer with an example.

OR

Can a body have constant speed and still be accelerating? Give an example.

OR

Explain how is it possible for an object to move with a constant speed but with uniform acceleration.

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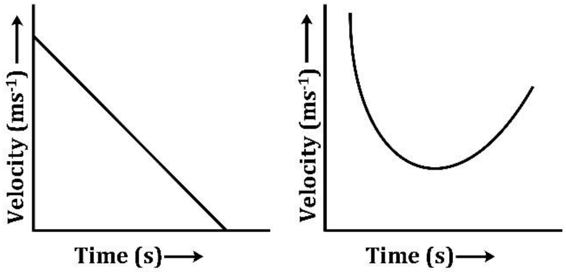
HOMEWORK

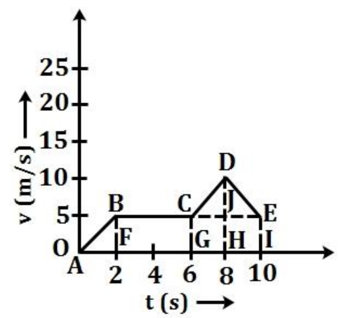
Q1. What do the graphs I) and II) shown here indicate?



I) II)

Q2. Find the total displacement of the body from the graph alongside.

Q3. The driver of train A travelling at a speed of 54 km/h applies brakes and retards the train uniformly. The train stops in 5 s. Another train B is travelling on the parallel with a speed of 36 km/h. Its driver applies the brakes and the train retards uniformly, train B stops in 10 s. Plot speed-time graphs for both the trains. Which train travelled farther after the brakes were applied?



Q4. The velocity-time graph for the motion of an object in a straight path is a straight line parallel to the time axis.

I. Identify the nature of motion of the body.
II. Find the acceleration of the body.
III. Draw the shape of distance-time graph for this type of motion.

Q5. Draw velocity-time graphs for the following cases:

I. When the object is at rest.
II. When the object is thrown vertically upwards.

Q6. A particle is moving with uniform positive acceleration. Its velocity-time graph will be

A) A straight line parallel to the time-axis
B) A straight line inclined at an obtuse angle to the time-axis
C) A straight line inclined at an acute angle to the time-axis
D) None of the above

Q7. Why is the motion of an athlete moving along the circular path, with a constant speed, an accelerated motion?

Q8. An artificial satellite is moving in a circular orbit of radius 42250 km. Calculate its speed if it takes 24 hours to revolve around the earth.

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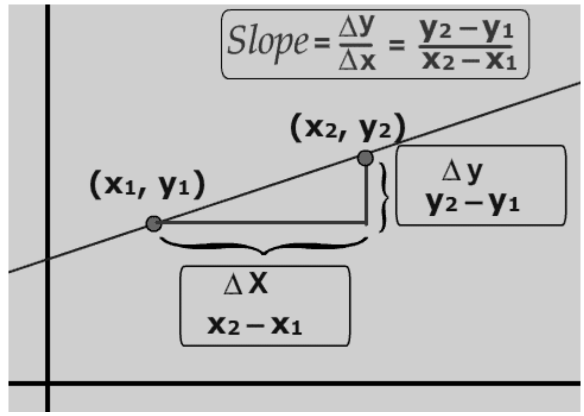
ADVANCED QUESTIONS

Slope of a Graph:

Every graph has two axes, usually represented by x axis and y axis. In this chapter, in most of the cases we plot time on the x axis and other quantities like displacement, velocity etc. on the y axis. One concept is particularly important for us, i.e slope.



Slope can be defined in many ways, simplest of them is that it is the ratio of change of quantity in y axis to that of the change in x axis.

Mathematically, Slope can be defined as $\frac{y_2-y_1}{x_2-x_1}$.

This may appear very unfamiliar, but we have learnt about it in this chapter.

Try recollecting the graph of displacement-time, in this graph we find velocity by using the formula $v = \frac{s_2-s_1}{t_2-t_1}$

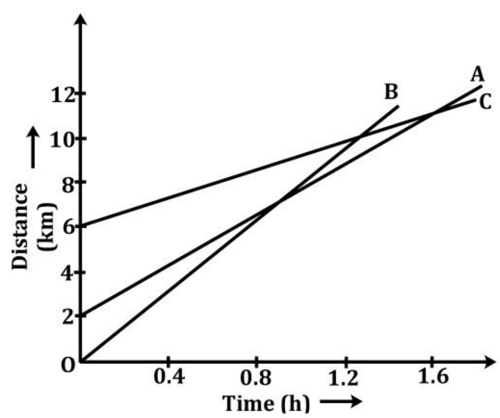
This is exactly in the same form as we have in the definition of slope. Hence we can conclude that velocity is the slope of displacement- time graph. By similar logic, we can say that speed is the slope of distance-time graph and acceleration is the slope of velocity-time graph.

Finding out the slope of a graph might appear a bit tedious to you, but there is an easy way out.

Try finding the angle the straight line graph makes with the positive x axis. If the angle made is acute, we call it a positive slope otherwise we call it a negative slope. For example, in the displacement-time curve if the angle made is obtuse, then it shows that acceleration is negative (retardation). We will study why exactly this happens in higher grades.

Another simple way of looking at a slope can be by finding the change in the quantity in y axis for a fixed/unit change in the quantity in x axis. The value of slope is proportional to the change in the corresponding y axis for a change in a unit quantity of x axis.

Q1. Figure shows the distance-time graph of three objects A, B and C. Study the graph and answer the following questions.



I. Which of the three is travelling the fastest?
II. Are all three ever at same point on the road?
III. How far has B travelled when it passes A?
IV. How far has B travelled by the time it passes C?

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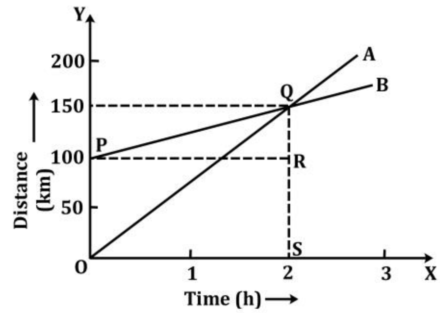
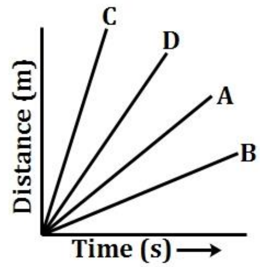
Q2. Figure shows the distance-time graphs of two trains, which start moving simultaneously in the same direction. From the graphs, find:

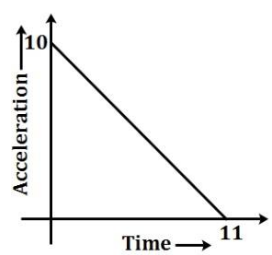
I. How much ahead of A is B when the motion starts?
II. What is the speed of B?
III. When and where will A catch B?
IV. What is the difference between the speeds of A and B?



Q3. Four cars A, B, C and D are moving on a levelled road. Their distance versus time graphs are shown in the figure. Which car is the slowest? Which is the fastest?

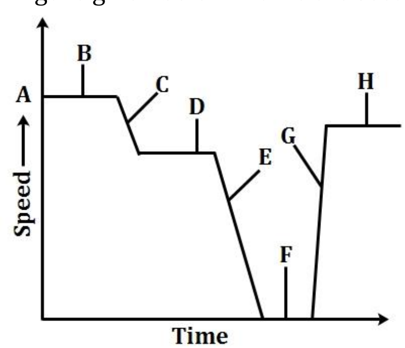


Q4. An article starts from rest. Its acceleration versus time t is as shown in the figure. The maximum speed of the particle will be



A) 110 m/s
B) 55 m/s
C) 10 m/s
D) Information insufficient

Q5. Look at the speed against time graph in figure given below. Name the sections that represent



I. Steady speed
II. Speeding up (accelerating)
III. Being stationary
IV. Showing down (decelerating)

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9P01.4 Equations of Motion

CONCEPTS COVERED

1. Significance of velocity-time, position-time and position-speed relations.
2. How to find the equations of motion using graphical method?
3. Solving various examples using the equations of motion.

IN CLASS EXERCISE

Q1. State the three equations of motion. Which of them describes

I. Velocity-time relation? II. Position-time relation?

Q2. Deduce the following equations of motion.

I. $s = ut + \left(\frac{1}{2}\right)at^2$ II. $v^2 = u^2 + 2as$

Q3. Brakes applied to a car produce an acceleration of 6 m/s^2 in the direction opposite to motion. If the car takes 2 s to stop after the application of brakes, calculate the distance it travels during this time.

Q4. A car starting from rest accelerates uniformly to acquire a speed 20km/h in 30 min. The distance travelled by car in this time interval will be:

A) 600 km B) 5 km C) 6 km D) 10 km

Q5. A trolley while going down an inclined plane has an acceleration of 2 cm/s^2. What will be its velocity 3 s after the start?

Q6. A racing car starting from rest has a uniform acceleration of 4 m/s^2. What distance will it cover in 10 s after the start?

Q7. A bus starting from rest moves with a uniform acceleration of 0.1 m/s^2 for 2 mins. Find

I. The speed acquired II. The distance travelled.

Q8. An electron moving with a velocity of $5 \times 10^4 m/s$ enters into a uniform electric field and acquires a uniform acceleration of 10^4 m/s^2 in the direction of its initial motion.

I. Calculate the time in which the electron would acquire a velocity double of its initial velocity.
II. How much distance would the electron cover in this time?

HOMEWORK

Q1. An object starts linear motion with a velocity 'u' and under uniform acceleration 'a' it acquires a velocity 'v' in time 't'. Draw its velocity-time graph. From this graph obtain the following equations:

I. $v = u + at$ II. $s = ut + \frac{1}{2}at^2$ III. $v^2 = u^2 + 2\,as$

Q2. Velocity of a train changes from 20 m/s to 25 m/s, when it accelerates at a rate 2 m/s^2. Find the distance covered by the train.

Q3. A moving train is brought to rest within 20 s by applying brakes. If the retardation due to brakes is 2 m/s^2, then the initial velocity was

A) 10 m/s B) 20 m/s C) 30 m/s D) 40 m/s

Q4. A train is travelling at a speed of 90 km/hr. Brakes are applied so as to produce a uniform acceleration of -0.5 m/s^2. Find how far the train will go before it is brought to rest.

Q5. A motorboat starting from rest on a lake accelerates in a straight line at a constant rate of 3 m/s^2 for 8 s. How far does the boat travel during this time?

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Q6. If the displacement of an object is proportional to square of time, predict the motion of the object.

Q7. A stone is thrown in a vertically upward direction with a velocity of 5 m/s. If the acceleration of the stone during its motion is 10 m/s^2 in the downward direction, what will be the height attained by the stone and what time will it take to reach there?

Q8. A bullet initially moving with a velocity of $20 m/s$ strikes a target and comes to rest after penetrating a distance $10 cm$ in the target. Calculate the retardation caused by the target.

Q9. Two stones are thrown vertically upwards simultaneously with their initial velocities u_1 and u_2, respectively. Prove that the heights reached by them would be in the ratio $u_1^2 : u_2^2$. (Assume upward acceleration is $-g$ and downward acceleration is $+g$).

ADVANCED QUESTIONS

Q1. A particle having initial velocity u is moving with a constant acceleration a for a time t.

I. Find the displacement of the particle in the last 1 s.
II. Evaluate the displacement of particle in the last 1 s when $u = 2\ m/s, a = 1\ m/s^2$ and $t = 5\ s$.

Q2. A body moving with a constant acceleration travels the distance $3m$ and $7m$ respectively in $1s$ and $3s$. Calculate

I. The acceleration of the body and
II. The initial velocity.

Q3. A bullet of mass $0.006 kg$ travelling at $120 m/s$ penetrates deeply into a fixed target and is brought to rest in $0.01 s$. Calculate the distance of penetration in the target.

Q4. A ball is tossed straight up from the ground. Half way up, it has a speed of $19.6 m/s$. Find

I. Its initial velocity
II. The maximum height reached

Q5. A particle experiences constant acceleration for 20 seconds after starting from rest. If it travels a distance s_1 in the first 10 seconds and distance s_2 in the next 10 seconds, then

A) $s_2 = s_1$ B) $s_2 = 2s_1$ C) $s_2 = 3s_1$ D) $s_2 = 4s_1$

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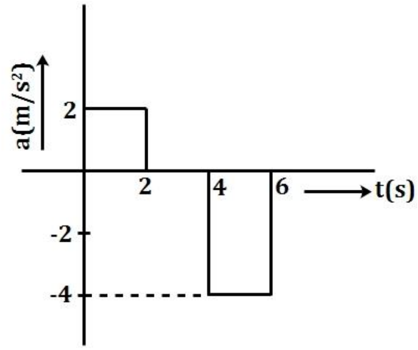
Advanced Practice Problems

Q1. The acceleration versus time graph of a particle moving along a straight line is shown in the figure. Draw the respective velocity – time graph. Given $v = 0$ at $t = 0$.



Q2. Which of the following statements are true for a moving body?

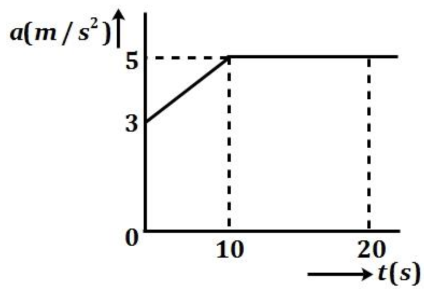
A) If its speed changes, its velocity must change and it must have some acceleration

B) If its velocity changes, its speed must change and it must have some acceleration

C) If it is accelerating, then its speed and velocity must change

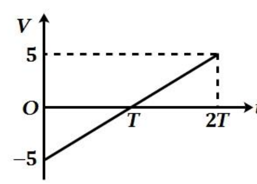
D) If its speed changes but direction of motion does not change, its velocity can remain constant

Q3. The graph describes an airplane's acceleration during its take – off run. The airplane's velocity when it lifts off at $t = 20s$ is:



A) $40m/s$ B) $50m/s$ C) $90m/s$ D) $180m/s$

Q4. The figure shows the velocity (v) of a particle plotted against time (t) Choose the correct alternative from the following.



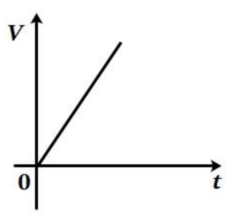
A) The particle changes its direction of motion at some point

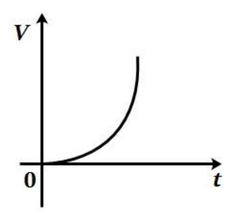
B) The acceleration of the particle remains constant

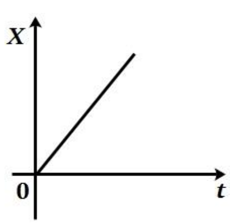
C) The displacement of the particle is zero

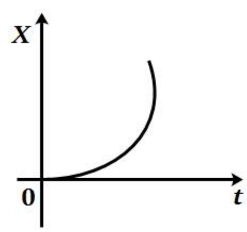
D) All of the above

Q5. Identify the correct graph representing the motion of a particle along a straight line with constant acceleration. (The curve in the 4^{th} option is an arc of a circle). More than one options can be correct in this question.

A) 

B) 

C) 

D) 

Q6. A body falls freely from a certain height. Show graphically the relation between the distance fallen and square of time. How will you determine g from this graph?

Q7. A ball moves on a smooth floor in a straight line with uniform velocity $10m/s$ for $6s$. At $t = 6s$, the ball hits a wall and comes back along the same line to the starting point with the same speed. Draw the velocity – time graph and use it to find the total distance travelled by the ball and its displacement.

Q8. A train starts from rest and accelerates uniformly at a rate of $2m/s^2$ for $10s$. It then maintains a constant speed for $200s$. The brakes are then applied and the train is uniformly retarded and comes to rest in $50s$. Find

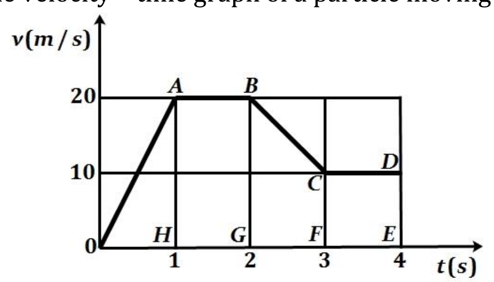
I. The maximum velocity reached,
II. The retardation in the last $50s$,
III. Total distance travelled, and
IV. The average velocity of the train

Q9. A particle starts with an initial velocity and passes successively over the two halves of a given distance with acceleration a_1 and a_2 respectively. Show that the final velocity is the same as if the whole distance is covered with a uniform acceleration $\frac{(a_1+a_2)}{2}$

Q10. A body moving along a straight line with uniform acceleration has a speed $15m/s$ when passing a point P and $35m/s$ when passing a point B in its path. Calculate its speed at the midpoint of PB.

Q11. A train, $100m$ long and stationary, is given the all clear by a signal $80m$ ahead of it. The train accelerates uniformly at $0.4m/s^2$. Find the time taken for the engine driver (at the front) and the guard (at the back) of the train to pass the signal. At what speed is the train moving at each of these times?

Q12. The following figure represents the velocity – time graph of a particle moving in a straight line



I. Did the particle ever move with uniform velocity?
II. Did the particle ever move with uniform acceleration?
III. What is the distance traversed by the particle in 4 seconds?
If your answer to any of the parts I or II is yes, then specify the region(s) and also specify/ calculate the corresponding value(s)

Solutions

9P01.1 DISTANCE, DISPLACEMENT & SPEED

IN CLASS EXERCISE

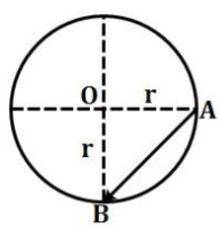
Q1. To describe the location of an object we need to specify its distance from a reference point, which we call the origin. For example, suppose that a library in a city is 2 km north of the railway station. We have specified the position of the library with respect to the railway station, i.e. in this case, the railway station acts as the reference point (origin).

Q2.

Distance	Displacement
1. Distance is the length of the actual path travelled by the object, irrespective of its direction of motion.	1. Displacement is the shortest distance between the initial and final positions of an object in a given direction.
2. Distance is a scalar quantity.	2. Displacement is a vector quantity.
3. Distance covered can never be negative. It is always positive or zero.	3. Displacement may be positive, negative or zero.
4. Distance between two given points may be different depending on the path chosen.	4. Displacement between two given points is always the same.

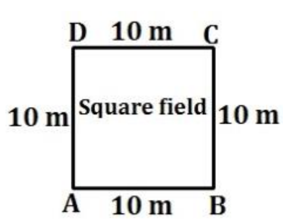
Q3. After completing half the circle, the particle will reach the diametrically opposite point, i.e. from point A to point B. The shortest distance between these points is the diameter of the circle, i.e. $2r$.

Q4. When the particle covers three quarters of a circle, the magnitude of its displacement is $AB = \sqrt{OA^2 + OB^2} = \sqrt{r^2 + r^2} = \sqrt{2}r$.



Q5. Farmer takes 40 s to move along the boundary of the square field, i.e. after 40 s the farmer is again at his initial position. Hence after every interval of 40 s his displacement becomes zero.

Time given = 2 min 20 s



$= (2 \times 60 + 20)s = 140\ s$

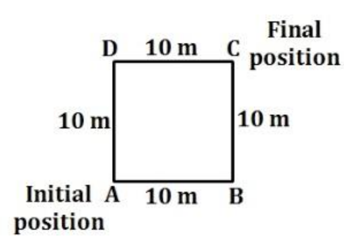
Displacement of farmer after 2 min 20 s, i.e. after 140 s

= Displacement after $(3 \times 40 + 20)s$

$= 0 +$ Displacement after 20 s

[∵ After each 40 s displacement is zero]

Farmer completes one round in 40 s, so he will complete half a round in 20 s, i.e. after 20 s final position of farmer would be C if he were to start at A.



Displacement of farmer, $AC = \sqrt{AB^2 + BC^2}$

$= \sqrt{10^2 + 10^2} = 10\sqrt{2}\ m$

$= 10 \times 1.414 = 14.14\ m.$

Q6. If an object covers equal distances in equal intervals of time, however small the time interval may be, then the object is said to be in uniform motion. For example, suppose a bus moves 10 km in the first 15 minutes, 10 km in second 15 minutes, 10 km in third 15 minutes and so on. Then one can say that the bus is in uniform motion.

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Q7. Average Speed is the total distance travelled by a body, divided by the total time taken to cover that distance. Thus,

Average Speed $= \dfrac{\text{Total distance travelled}}{\text{Total time taken}}$

For example, if a car travels a distance of 100 km in 2 hours, then its average speed

$= \frac{100\ km}{2\text{ hour}} = 50\ km$ per hour.

Q8. Let the distance one way $= x\ km$

Time taken in the forward journey at a speed of $50\ kmph = \frac{\text{Distance}}{\text{Speed}} = \frac{x}{50}h$

Time taken in the return journey at a speed of $40\ kmph = \frac{x}{40}\text{hr}$

Total time for the whole journey

$= \left(\dfrac{x}{50} + \dfrac{x}{40}\right)h = \left(\dfrac{4x+5x}{200}\right)h = \dfrac{9x}{200}h$

Total distance covered $= 2x$

$\therefore$ Average Speed $= \frac{\text{Total distance}}{\text{Total time}}$

$= \dfrac{2x\ km}{\frac{9x}{200}\ h} = \dfrac{\left(\frac{2x\times 200}{9x}\right)km}{h}$

$= \left(\frac{400}{9}\right) km/h = 44.44\ km/h.$

Q9. $v_{av} = \frac{s_1+s_2}{t_1+t_2} = \frac{s}{\frac{s_1}{v_1}+\frac{s_2}{v_2}}$

Or $40\ km/h = \frac{100\ km}{\frac{30}{30}+\frac{70}{v_2}\ h}$

Or $1 + \frac{70}{v_2} = \frac{100}{40}$

Or $\frac{70}{v_2} = \frac{5}{2} - 1 = \frac{3}{2}$

Or $v_2 = \frac{\frac{70\times 2}{3}km}{h}$

$= \dfrac{140}{3}\dfrac{km}{h}$

$= 46.7\ km/h.$

HOMEWORK

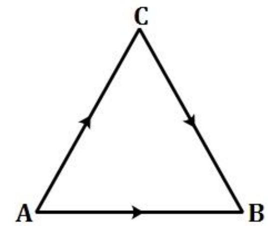
Q1. Distance: It is the length of the actual path travelled by a body between its initial and final positions. In the figure, suppose a body moves from position A to B through C. Then,

Distance travelled $= AC + BC$

Distance is a scalar quantity because it has only magnitude and no direction. Distance covered is always positive or zero. Distance describes the total path covered by an object.

Displacement: The change in the position of an object in a given direction is known as displacement. It is the shortest distance measured in the direction from the initial to the final position of the body. In the figure,

Displacement $= \overrightarrow{AB}$



As displacement has both magnitude and direction, so it is a vector quantity. If the measurement of displacement is expressed without direction, it is known as the *magnitude of displacement*. The magnitude of displacement is always less than or equal to the distance travelled.

Displacement may be positive, negative or zero. Displacement is used to locate the final position of an object with reference to its initial position at a given time.

Q2. I. Distance $= \frac{1}{2}\times$Circumference of the circle

$= \dfrac{1}{2}\times 2\pi r = \pi r$

Displacement $= 2r$

II. Distance $= 2\pi r$

Displacement $= 0$ [Since initial and final positions are the same]

Q3. Distance covered after 2 complete rounds

$= 2\times$ Circumference

$= 2\times 2\pi(2R) = 8\pi R$

After 2 complete rounds, the body comes back to its initial position. $\therefore$ Displacement$= 0$.

Q4. I. Diameter of circular track $= 200\ m$

$\therefore$ Radius of circular track

$= \dfrac{200}{2}m = 100\ m$

Circumference of circular track $= 2\pi r$

$= \left(2\times\frac{22}{7}\times 100\right) m = \frac{4400}{7}m$

Total time given $= 2$ min $20s$

$= 2\times 60 + 20 = 140\ s$

According to question, athlete takes 40 s to complete one round.

$\therefore$ Distance covered in $40\ s = \frac{4400}{7}m$

Distance covered in $1\ s = \frac{4400}{7\times 40}m$

So distance covered in 140 s

$\Rightarrow \left(\dfrac{4400}{7\times 40}\times 140\right)m$

$\Rightarrow 2200\ m = 2.2\ km$

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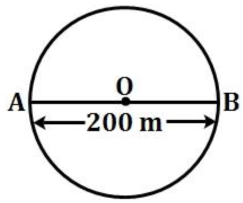
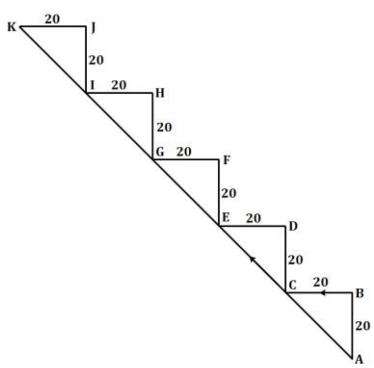
II. As per the question, athlete completes one round in $40\ s$, i.e. after $40\ s$ athlete comes back to its initial position or after $40\ s$ his displacement is zero. Similarly, after $120\ s\ (40 \times 3)$, his displacement is zero.
$\therefore$ Displacement after 2 min $20\ s$ or $140\ s$
$=$ Displacement after $20\ s$
In $20\ s$ athlete will cover half the circular track from A to B (in $40\ s$ one round is completed, so in $20\ s$ half of the round will be completed.)



Displacement after $20\ s = AB$
$=$ Diameter of circular track$= 200\ m$.
(Initial position is A and after half round, final position is B)

Q5. Distance covered by ant $= AB + BC + CD + DE + EF + FG + GH + HI + IJ + JK$
$= 5 \times (20\ cm + 20\ cm)$
$= 5 \times 40 = 200\ cm$



Displacement $= AK = 5 \times \sqrt{(20)^2 + (20)^2}$
i.e. sum of all hypotenuse of the right angled triangles $= 5 \times \sqrt{800} = 100\sqrt{2}m$.

Q6. If an object covers unequal distances in equal intervals of time, it is said to be in non-uniform motion. Most of the motions seen in our daily life are non-uniform.
Some examples of non-uniform motion are:
I. A stone dropped from the top of a building.
II. A ball thrown vertically upwards.
III. The motion of a train as it leaves the station.
IV. The motion of a bus as it approaches a bus-stop.
V. The motion of a ball rolling down an inclined plane.

Q7. No, the speedometer of a car does not measure its average speed. It measures the speed at every instant, which is called the instantaneous speed.

Q8. Let the distance of school from Abdul's home be $x\ km$.
Time taken by Abdul to reach the school
$$= \frac{\text{Distance}}{\text{Average speed}} = \frac{x\ \ km}{20\ \ km/hr}\ h$$
Similarly, time taken by Abdul to reach the home from school
$$= \frac{\text{Distance}}{\text{Average speed}} = \frac{x\ \ km}{40\ km/hr}\ h$$
Average speed for Abdul's entire trip
$$= \frac{\text{Total distance}}{\text{Total time}} = \frac{x + x\quad km}{\frac{x}{20} + \frac{x}{40}\quad hr}$$
$$= \frac{2x}{\frac{2x+x}{40}} = \frac{2x \times 40}{3x}\frac{km}{h} = \frac{80}{3}km/h$$

Q9. Average speed, $v_{av} = 12\ km/h$
$$v_{av} = \frac{s_1 + s_2}{t_1 + t_2} = \frac{v_1 t_1 + v_2 t_2}{t_1 + t_2}$$
$$12 = \frac{9 \times \frac{10}{60} + v_2 \times \frac{20}{60}}{\frac{10}{60} + \frac{20}{60}}$$
Thus, $6 = 1.5 + \frac{v_2}{3}$
$\Rightarrow v_2 = 4.5 \times 3 = 13.5\ km/h$.

ADVANCED QUESTIONS

Q1. C
Assume the train is about to enter the platform. If the train travels $500\ m$, the front of the train will reach the end of the platform. For the train to completely pass the platform, it must travel another $100\ m$ so that its last compartment clears the platform. Total length of path covered by train
$= 500 + 100 = 600\ m$
Speed of train $= 20\ m/s$
Time taken by train to cross the bridge
$$= \frac{\text{Distance}}{\text{Speed}}$$
$$= \frac{600\ m}{20\frac{m}{s}}$$
$= 30\ s$

Q2. Let the whole journey be of distance D.
Part 1: (One third with speed u)
Distance $=$ D/3; Speed $=$ u
Time $=$ Distance/Speed $=$ D/3u.
Part 2: (One third with speed v)
Distance $=$ D/3; Speed $= v$
Time $=$ Distance/Speed $= D/3v$.
Part 3: (One third with speed w)
Distance $=$ D/3; Speed $= w$
Time $=$ Distance/Speed $= D/3w$.
Average speed over entire journey
$$= \frac{Total\ Distance}{Total\ Time} = \frac{D}{\frac{D}{3u}+\frac{D}{3v}+\frac{D}{3w}}$$
$$= \frac{3uvw}{uv+vw+uw}$$

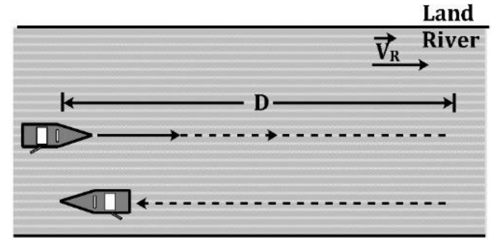
Q3. When two bodies with speeds v_1 and v_2 (assume $v_1 > v_2$) respectively are in motion, we can find the relative motion between them:

I. When they are in the same direction, $v_{rel.} = v_1 - v_2$
II. When they are in opposite directions, $v_{rel.} = v_1 + v_2$

Let u and v be the speeds of the two bodies respectively.
According to the question,
$u + v = 8$ and $u - v = 4$
Solving the above equations, we get
$2u = 12\ m/s \Rightarrow u = 6\ m/s$
And $v = 8 - u = (8-6)\ m/s$
$= 2\ m/s$.

Q4. Let $\vec{v}_R$ and $\vec{v}_B$ be the velocities of the river current and boat relative to the river bank. Then the velocity of the boat with respect to the water is $\vec{v}_{BR} = \vec{v}_B - \vec{v}_R$

$\therefore\ \vec{v}_B = \vec{v}_{BR} + \vec{v}_R$



Boat heading downstream for a distance *D* and returning to its starting point

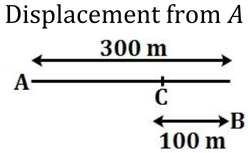
Data:
$v_{BR} = 1.8\ kmph, v_R = 0.9\ kmph$
We take the direction of the river current as the positive direction
Then, while going downstream,
$v_B = 1.8 + 0.9 = 2.7\ kmph$
So that the time taken to row downstream is
$$t_D = \frac{D}{v_B}$$
$$= \frac{2.7}{2.7} = 1h$$
While returning upstream, the velocity of the boat relative to water is $-1.8\ kmph$ (minus sign implies it is in the opposite direction). Hence, the velocity of the boat relative to the river bank and the time taken for the return journey are, respectively,
$v_B = -1.8 + 0.9$
$= -0.9\ kmph$
And $t_u = \frac{D}{v_B}$
$$= \frac{2.7}{0.9} = 3\ h$$
$\therefore$ The time for the round trip is
$t_d + t_u = 4\ h$

9P01.2 VELOCITY & ACCELERATION

IN CLASS EXERCISE

Q1. It is a physical quantity that gives both the speed and direction of motion of the body.
Velocity of a body is defined as the displacement produced per unit time. It is also defined as the speed of a body in a given direction.
$$\text{Velocity} = \frac{\text{Displacement}}{\text{Time}}$$
If s is the distance travelled by a body in a given direction and t is the time taken to travel that distance, then the velocity v is given by
$$v = \frac{s}{t}$$
The SI unit of velocity is m/s.
Velocity is a vector quantity because its description requires both magnitude and direction.

Q2. Distance covered from A to $B = 300\ m$
Displacement from A to $B = 300\ m$



[∵ If an object is moving along a straight line, then its distance and displacement are same]
Time taken by Joseph to move from A to B
$t = 2$ min $30\ s = 2 \times 60 + 30$
$= 120 + 30 = 150\ s$ [∵ 1 min= 60 s]

I. Average speed in jogging from A to B $= \frac{\text{Distance from A to B}}{\text{Time taken from A to B}} = \frac{300\ m}{150\ s} = 2\ m/s$
Average velocity in jogging from A to B $= \frac{\text{Displacement from A to B}}{\text{Time taken from A to B}} = \frac{300\ m}{150\ s} = 2\ m/s$

II. Distance covered from A to $C = AB + BC$
$= 300 + 100 = 400\ m$
Displacement from A to C = minimum distance from A to C
$= 300 - 100 = 200\ m$
Time taken in jogging from A to C
$= 2$ min $30\ s + 50\ s$
$= 2 \times 60 + 30 + 50 = 200\ s$
Average speed in jogging from A to C
$= \frac{\text{Displacement from A to B}}{\text{Time taken from A to B}}$
$= \frac{400\ m}{200\ s} = 2\ m/s$
Average velocity in jogging from A to C
$= \frac{\text{Displacement from A to C}}{\text{Time taken from A to C}}$
$= \frac{200\ m}{200\ s} = 1\ m/s$

Q3. Acceleration is defined as the rate of change of velocity with time, i.e.
Acceleration $= \frac{\text{Change in velocity}}{\text{Time taken}}$
$= \frac{(\text{Final velocity} - \text{Initial velocity})}{\text{Time taken}}$
If the velocity of a body changes from u to v in time t, then mathematically, acceleration a is given by $a = \frac{(v-u)}{t}$.

Q4. I. A body is in uniform acceleration if it travels in a straight path and its velocity increases or decreases by equal amounts in equal time intervals.
II. A body is in non-uniform acceleration if it travels in a straight path when its velocity increases or decreases by unequal amount in equal time intervals.

Q5. Given, final velocity, $v = 0, a = -3\ m/s^2$
Time taken, $t = 10\ s$
As $a = \frac{v-u}{t}$
$u = v - at = 0 - (-3)(10)\ m/s = 30\ m/s$.

Q6. I. When a body falls freely towards the earth, the acceleration due to gravity acts in its direction of motion (downward).
II. When a body is thrown up, the acceleration due to gravity acts against its direction of motion.
III. A body falling freely towards the earth has a uniform acceleration of 9.8 m/s^2.
IV. When a ball is thrown on a rough surface, it has non-uniform acceleration.

Q7. B
Initial velocity, $u = 5\ m/s$
Acceleration, $a = 2\ m/s^2$
Time, $t = 10\ s$
Final velocity, $v =?$
As $a = \frac{v-u}{t}$
$v = u + at \Rightarrow v = 5 + 2 \times 10 = 25\ m/s$.

Q8. I. A particle thrown upwards has its velocity in opposite direction to its acceleration (g, downwards)
II. When the particle is released from rest from a certain height, its velocity is zero while acceleration is g downwards. Similarly, at the extreme position of a pendulum velocity is zero, while acceleration is not zero
III. In uniform circular motion velocity is perpendicular to its radial or centripetal acceleration

HOMEWORK

Q1. Suppose the insect was at A initially and it moved along ACB to reach the diametrically opposite point B in 1 min.
I. The distance moved in 1 min $= \pi r$
$= (3.14 \times 10)\ cm = 31.4\ cm$
II. Speed $= \frac{\text{Distance}}{\text{Time}} = \frac{31.4}{1} = 31.4\ cm/\text{min}$
III. Displacement, $AB = 2r$
$= (2 \times 10)\ cm = 20\ cm$
IV. Average velocity,
$v_{av} = \frac{\text{Displacement}}{\text{Time}} = \frac{20\ cm}{1\ \text{min}} = 20\ cm/\text{min}$
In the direction A to B.

Q2. I. Average speed$= \frac{s_1+s_2}{t_1+t_2}$
$= \frac{(30+30)\ m}{(4+2)\ s} = 10\ m/s$.
II. No, the direction of motion of the object is also needed for finding its average velocity.
III. The two magnitudes will be equal when the motion is only in one particular direction and the direction should not change during motion.

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Q3. The relationship between u, v, a and t is
$a = \frac{v-u}{t}$
A body falling freely towards the earth has a uniform acceleration of 9.8 m/s^2.

Q4. The motion of the particle is said to be accelerated if the velocity of the particle changes with time. When the instantaneous speed of the particle decreases with time, the acceleration is opposite to the instantaneous velocity and is called retardation or deceleration.

Q5. A
$a = \frac{v-u}{t} \Rightarrow u = v - at$
$u = 0 - (-1.5 \times 20)$
$u = 30\ m/s$

Q6. D
Converting velocities to m/s,
we have $u = \frac{100}{18}\ m/s$ and $v = \frac{250}{18}\ m/s$.
$a = \frac{v-u}{t} = \frac{150}{180}$
$= \frac{5}{6} = 0.83\ m/s^2$

Q7. Initial velocity, $u = 0\ m/s$
Final velocity, $v = 54\ km/h = 15\ m/s$

Acceleration, $a = 3\ m/s^2$
So, Time $t = \frac{v-u}{a}$
$= \frac{15-0}{3} = 5s$

Q8. In a given time interval, the displacement and average velocity of a particle are both zero if its initial and final positions are the same, i.e., when the particle is at rest or it returns to its initial position.
For a particle at rest, the path length is zero and hence, the average speed is also zero. In the second case, the total path length > 0 so that
Average speed $= \frac{\text{total path length}}{\text{elapsed time}} > 0$

Q9. D

ADVANCED QUESTIONS

Q1. A deceleration can reduce the velocity of a particle to zero and then increase it in a direction opposite to its initial velocity. At the turning point of the motion, the particle has an acceleration but its instantaneous velocity is zero.
For example, when a particle is thrown straight up, the Earth's gravitational force downward decelerates the particle until its velocity becomes momentarily zero. At this peak point of its motion, the particle has zero instantaneous velocity but constant downward acceleration.

Q2. Let the path length be x and the time taken be t.

For car A:

Final velocity (v) – Initial velocity $(u) = at$
$v = 4 + 1 \times t$
Distance = Average speed $\times$ time
$= \frac{\mathrm{u+v}}{2} \times \mathrm{t}$
$= \frac{4 + (4+\mathrm{t})}{2} \times \mathrm{t}$
$= \frac{8\mathrm{t} + \mathrm{t}^2}{2}$
[Note: Distance is given by average velocity × time only when acceleration is constant. Also, for constant acceleration, average velocity $= (v + u)/2$]
For Car B:
Final velocity (v) – Initial velocity $(u) = at$
$v = 2 + 2 \times t$
Distance = Average speed $\times$ time
$= \frac{\mathrm{u+v}}{2} \times \mathrm{t} = \frac{2 + (2+2\mathrm{t})}{2} \times \mathrm{t}$
$= \frac{4\mathrm{t} + 2\mathrm{t}^2}{2} = 2\mathrm{t} + \mathrm{t}^2$
Since, both the distances are same, on equating both of them, we get
$\frac{8\mathrm{t} + \mathrm{t}^2}{2} = 2\mathrm{t} + \mathrm{t}^2$
$t^2 - 4t = 0$
On solving, $\mathrm{t} = 0, 4\mathrm{s}$ {Ignoring the $t = 0s$ as at $t = 0s$, motions starts and both are at the initial position}
Putting $t = 4s$ in any of the above equations, we get
$x = 2 \times 4 + 4^2 = 24$ m, which is the length of the path

Q3. Average acceleration $= \frac{\text{change in velocity}}{\text{time taken}}$

Initial velocity $= 25\ m/s$
Final velocity $= -24\ m/s$ {negative direction because of opposite direction}
$\Rightarrow$ Average acceleration $= \frac{(-24)-(25)}{3.50 \times 10^{-3}}$
$= -\frac{49 \times 10^3}{3.5} m/s^2 = -1.4 \times 10^4\ m/s^2$
Magnitude $= 1.4 \times 10^4\ m/s^2$

Soln.

Q4. Let the acceleration $= a$.

Initial velocity $= u$
Final velocity, $v = u + at$
In this time interval, average velocity,
$$v_{avg} = \frac{u+v}{2}$$
$$= u + \frac{at}{2}$$
This can be greater or smaller than the final velocity, it may also be half the final velocity. Hence we cannot be sure about options A, B and D.
In an accelerated motion, average velocity may be zero as well.
Hence the correct option is (C).

Q5. Velocity of police jeep $= 72\ km/h\ = 20\ m/s$

Velocity of thief's jeep $= 90\ km/h = 25\ m/s$

Velocity of bullet (with respect to the jeep) $=$ $180\ m/s$

Now, since bullet is fired from police jeep which is going at $20\ m/s$,

Therefore, velocity of bullet is

$(180 + 20)\ m/s = 200\ m/s$.

To calculate velocity with which bullet with hit the thief we use concept of relative velocity.

Therefore, we have $V_{BT} = V_B - V_T$

Here V_B is velocity of bullet and V_T is velocity of thief

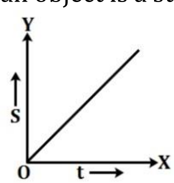
$V_{BT} = 200\ m/s\ - 25\ m/s\ = 175\ m/s$

As the thief is 350 m away from the police jeep at the time when the bullet is fired, time taken would be $(350/175)\ s = 2\ s$.

9P01.3 GRAPHICAL REPRESENTATION OF MOTION & UNIFORM CIRCULAR MOTION

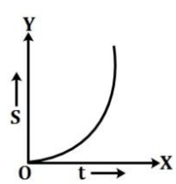
IN CLASS EXERCISE

Q1. The distance time graph for uniform motion of an object is a straight line.

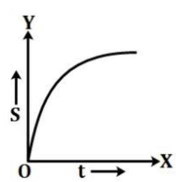


Uniform motion

The distance time graph for non-uniform motion of an object is a curved line.



(Speed increases)

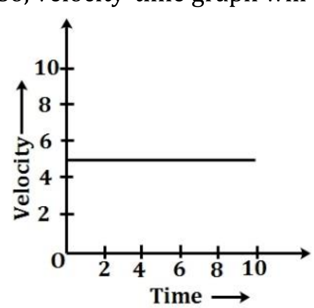


(Speed decreases)

Non – uniform motion

Q2. From the given v-t graph, it is clear that the velocity of the object is not changing with time, i.e. the object is in uniform motion.

Q3. From the graph,
Velocity after $2\ s = \frac{10}{2}\ m/s = 5\ m/s$
Velocity after $4\ s = \frac{20}{4}\ m/s = 5\ m/s$
Velocity after $6\ s = \frac{30}{6}\ m/s = 5\ m/s$
E.g. whenever velocity is constant, acceleration $= 0$
So, velocity-time graph will be drawn as



Q4. By measuring the area under v-t curve.

Q5. Displacement = Area under v-t curve
$= \frac{1}{2}$ (Sum of parallel sides) ×height
$= \frac{1}{2}(AD + BC) \times CF$
$= \frac{1}{2}(10 + 6) \times 12.5\ \ m = 100\ m$

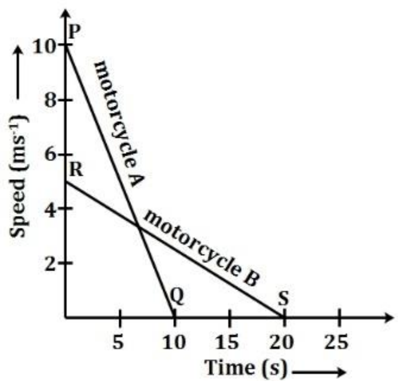
Q6. I. From the graph, it is clear that velocity is not changing with time, i.e. acceleration is zero.
II. Again from the graph, we can see that there is no change in the velocity with time, so

Soln.

velocity after 15 s will remain the same as 20 m/s.

III. Distance covered in 15 s
$= \text{velocity} \times \text{time} = (20 \times 15)\ m$
$= 300\ m$.

Q7. Here $u_A = 36\ km/h = 36 \times \frac{5}{18} m/s = 10\ m/s$, $v_A = 0, t_A = 10\ s$
$u_B = 18\ km/h = 5\ m/s,\ v_B = 0, t_B = 20\ s$
The speed-time graph for the two motorcycles are shown below:



Distance travelled by motorcycle A,
s_A =Area under $PQ = \frac{1}{2} \times 10 \times 10 = 50\ m$
Distance travelled by motorcycle B
s_B =Area under $RS = \frac{1}{2} \times 20 \times 5 = 50\ m$
Thus both the motorcycles travelled the same distance before coming to a stop.

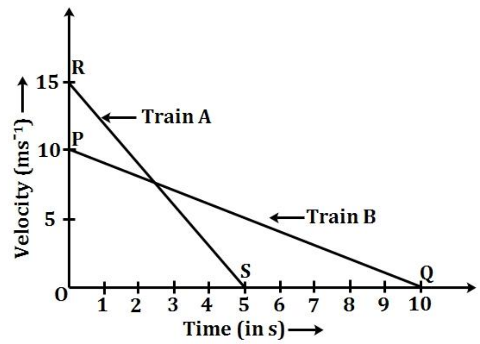
Q8. An object moving with constant speed can be accelerated if its direction of motion changes. For example, an object moving with a constant speed in a circular path has an acceleration because its direction of motion changes continuously.

HOMEWORK

Q1. Graph I) indicates that a body has certain initial speed and then its speed uniformly decreases and finally becomes zero.
Graph II) shows that a body first retards and then accelerates.

Q2. Total displacement
= Area under the velocity-time graph
= Sum of areas (ΔABF +rectangle $BCGF$ + ΔCDE +rectangle $CEIG$)
$= \left[\frac{1}{2}(AF \times FB) + (BC \times CG) + \frac{1}{2}(CE \times DJ) + (CG \times GI)\right]$
$= \left[\frac{1}{2}(2 \times 5) + (4 \times 5) + \frac{1}{2}(4 \times 5) + (5 \times 4)\right]\ m = 55\ m$.

Q3. For train A, the initial velocity,
$u = 54 \frac{km}{hr}$
$= 54 \times \frac{5}{18} \frac{m}{s}$
$= 15\ m/s$
Final velocity, $v = 0$ and time, $t = 5\ s$
For train $B, u = 36 \frac{km}{hr}$
$= 36 \times \frac{5}{18} = 10\ m/s$
$v = 0,\ t = 10\ s$
Speed-time graph for train A and B are shown in figure below:

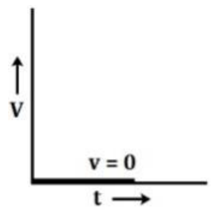


Distance travelled by train A = Area under RS
= Area of $\Delta ORS = \frac{1}{2} \times OR \times OS$
$= \left(\frac{1}{2} \times 15 \times 5\right) m = 37.5\ m$
Distance travelled by train B = area under PQ
$= \frac{1}{2} \times OP \times OQ$
$= \left(\frac{1}{2} \times 10 \times 10\right) m = 50\ m$
So, train B travelled farther after the brakes were applied.

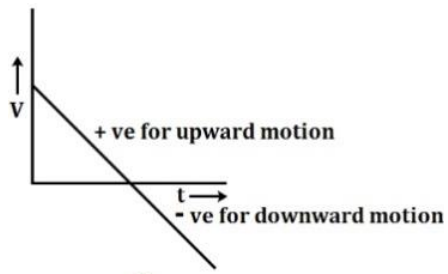
Q4. I. The body is moving with a uniform velocity.
II. Acceleration = 0.
III. The distance-time graph will be a straight line inclined to the time-axis as shown in Fig.

Q5. I. When the object is at rest, $v = 0$. The v-t is a straight line on the time-axis, as shown in Fig (a)

II. When the object is thrown vertically upwards, it has decreasing velocity for upward motion and then increasing velocity for downward motion, as shown in Fig. (b).



a)



b)

Q6. C

Q7. The motion is an accelerated motion because the direction of motion of the athlete changes continuously at every point of the circular path.

Q8. Here, $r = 42{,}250\ km$

$T = 24\ h$

Total path length $= 2\pi r$

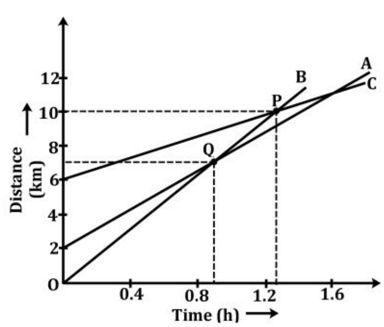
Speed, $v = \frac{2\pi r}{T}$

$= \dfrac{2 \times 3.14 \times 42{,}250}{24} kmph$

$= 11055\ kmph$

ADVANCED QUESTIONS

Q1.



I. The object for which slope of speed-time graph is maximum will have maximum speed, i.e., will travel the fastest. Here for object B, slope is maximum so it is travelling the fastest.

II. All the three objects will be at the same point on the road if the speed-time graph intersect each other at any point. Here all the three graphs do not intersect each other, so these three will never be at the same point on the road.

III. When B passes A, it travels a distance of $7\ km$

IV. Distance travelled by C by the time it passes $B = 10\ km$.

Q2. I. B is ahead of A by distance $OP = 100\ km$ when the motion starts.

II. Speed of $B = \frac{QR}{PR}$

$= \dfrac{(150 - 100)\ km}{(2 - 0)\ h} = 25\ km/h$

III. As it is clear from graphs, A will catch B at point Q, i.e. after 2 hours and at a distance of $150\ km$.

IV. $\because$ Speed of $A = \frac{QS}{OS}$

$= \dfrac{(150 - 0)\ km}{(2 - 0)\ h} = 75\ km/h.$

$\therefore$ Difference in speeds $= 75 - 25 = 50\ km/h.$

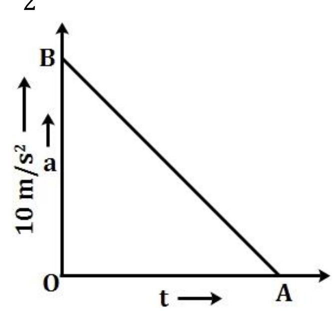
Q3. The slope of distance-time graph represents the speed. From the graph, it is clear that the slope of distance-time graph for car B is minimum. So car B is the slowest. By the same logic, C is the fastest.

Q4. B

The area under acceleration-time graph gives change in velocity. As acceleration is zero at the end of $11\ s$.

$\therefore v_{max} =$ Area of $\angle OAB$

$= \dfrac{1}{2} \times 11 \times 10 = 55\ m/s$



Q5. The graph illustrates the below points:

C, E →Sloping downwards. The speed of the object is decreasing.

B, D, F, H →Horizontal. The speed of the object is constant.
G → Sloping upwards. The speed of the object is increasing.

I. B, D and H have steady speed.
II. G is speeding up or accelerating.
III. F is being stationary.
IV. C and E are slowing down i.e. decelerating.

9P01.4 EQUATIONS OF MOTION

IN CLASS EXERCISE

Q1. For a body moving along a straight line at velocity u and accelerating uniformly with a for time t to attain a velocity v and cover displacement s. Thus, the three equations of motion are as follows:

First equation, $v = u + at$

Second equation, $s = ut + \frac{1}{2}at^2$

Third equation, $v^2 = u^2 + 2as$

I. First equation represents velocity-time relation.

II. Second equation represents position-time relation.

Q2. I. Consider a body which starts with initial velocity u and due to uniform acceleration a, its final velocity becomes v after time t. Then, its average velocity is given by

Average velocity

$= \frac{\text{Initial velocity} + \text{Final velocity}}{2} = \frac{u+v}{2}$

∴The distance covered by the body in time t is given by

Distance, $s =$Average velocity × Time

Or $s = \frac{u+v}{2} \times t \Rightarrow s = \frac{u+(u+at)}{2} \times t$

$\therefore s = \frac{2\,ut+at^2}{2}$ or $s = ut + \frac{1}{2}at^2$

II. We know that,

$s = ut + \frac{1}{2}at^2$ (1)

Also, $a = \frac{v-u}{t}$

$\Rightarrow t = \frac{v-u}{a}$

Substituting the value of t in Eq. (1), we have

$s = u\left(\frac{v-u}{a}\right) + \frac{1}{2}a\left(\frac{v-u}{a}\right)^2$

Or $s = \frac{uv - u^2}{a} + \frac{v^2 + u^2 - 2\,uv}{2a}$

Or $2\,as = 2\,uv - 2u^2 + v^2 + u^2 - 2\,uv$

Or $v^2 - u^2 = 2\,as$

Or $v^2 = u^2 + 2\,as$.

Q3. Final velocity, $v = 0$

[Since, car stops after applying the brakes]

$a = -6\ m/s^2,\ t = 2\ s$

From first equation of motion, $v = u + at$

Or $u = v - at = 0 - (-6)(2) = 12\ m/s$

From second equation of motion,

$s = ut + \frac{1}{2}at^2$

$= (12)(2) + \frac{1}{2}(-6)(2)^2\ \ m = 12\ m.$

Q4. $u = 0$

$t = 30min = \frac{30}{60}h = \frac{1}{2}h$

Using $v = u + at$, we have

$20 = 0 + 1 \times \frac{1}{2} = \frac{a}{2}$

$\therefore a = 40km/h$

Again using $2as = v^2 - u^2$, we have

$2 \times 40 \times s = (20)^2 - (0)^2$ or $80s = 400$

$\therefore s = \frac{400}{80} = 5km$

Q5. Given, initial velocity of trolley, $u = 0$

Acceleration, $a = 2\ cm/s^2 = 0.02\ m/s^2$

Time taken, $t = 3\ s$

From first equation of motion,

$v = u + at = (0 + 0.02 \times 3)\ m/s = 0.06\ m/s$

Hence, the velocity of the trolley $3\ s$ after the start is $0.06\ m/s$ or $6\ cm/s$.

Q6. Given acceleration,$a = 4\ m/s^2$

Time taken, $t = 10\ s$,

Initial velocity of the racing car, $u = 0$

From second equation of motion,

$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2} \times 4 \times (10)^2\ \ m$

$= 200\ m.$

The distance travelled by the racing car in $10\ s$ is $200\ m$.

Q7. Here, acceleration, $a = 0.1\ m/s^2$

Time, $t = 2$ mins $= (2 \times 60)\ s = 120\ s$

Initial speed, $u = 0$

Soln.

I. From first equation of motion,
Speed acquired by the bus,
$v = u + at = (0 + 0.1 \times 120)\frac{m}{s}$
$= 12\ m/s$

II. From second equation of motion,
Distance travelled, $s = ut + \frac{1}{2}at^2$
$= 0 \times 120 + \frac{1}{2} \times 0.1 \times 120 \times 120$
$= 0.1 \times 60 \times 120 = 720\ m$
The distance travelled by the bus is $720\ m$.

Q8. Given, $u = 5 \times 10^4 m/s, a = 10^4 m/s^2$
$v = 2u = 10^5 m/s$
From first equation of motion, $v = u + at$

I. $t = \frac{v-u}{a} = \frac{10^5-(5\times10^4)}{10^4} = \frac{5\times10^4}{10^4} = 5\ s$
From third equation of motion,
$v^2 = u^2 + 2as$

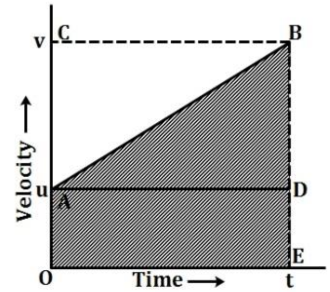
II. $s = \frac{v^2-u^2}{2a} = \frac{10^{10}-(25\times10^8)}{2\times10^4}$
$= \dfrac{75 \times 10^8}{2 \times 10^4} = 3.75 \times 10^5\ m.$

HOMEWORK

Q1. Equations of motion by graphical method: Consider an object moving along a straight line with initial velocity u and uniform acceleration a. Suppose, it has a displacement s in time t. As shown in Fig, its velocity-time graph is a straight line. Here $OA = ED = u, OC = EB = v$ and $OE = t = AD$.

I. Equation for velocity-time relation: We know that
Acceleration = Slope of velocity-time graph AB
or $a = \frac{DB}{AD} = \frac{DB}{OE} = \frac{EB-ED}{OE} = \frac{v-u}{t}$
or $v - u = at$
or $v = u + at$
This proves the first equation of motion.

II. Equation for position-time relation.
From the above part, we have
$a = \frac{DB}{AD} = \frac{DB}{t}$
Or $DB = at$



Displacement of the object in time t is
$s =$ Area of the trapezium $OABE$
=Area of rectangle $OADE$ +Area of triangle ADB
$= OA \times OE + \frac{1}{2}DB \times AD$
$= ut + \frac{1}{2}at \times t$
Or $s = ut + \frac{1}{2}at^2$
This proves the second equation of motion.

III. Equation for position-velocity relation: The displacement of the object in time t is
$s =$ Area of the trapezium $OABE$
$= \frac{1}{2}(EB + OA) \times OE = \frac{1}{2}(EB + ED) \times OE$
Acceleration, $a =$ Slope of velocity-time graph AB
Or $a = \frac{DB}{AD} = \frac{EB-ED}{OE}$ or $OE = \frac{EB-ED}{a}$
$\therefore s = \dfrac{1}{2}(EB + ED) \times \dfrac{(EB - ED)}{a}$
$= \dfrac{1}{2a}(EB^2 - ED^2) = \dfrac{1}{2a}(v^2 - u^2)$
Or $v^2 - u^2 = 2as$
This proves the third equation of motion.

Q2. Initial velocity, $u = 20\ m/s$
Final velocity, $v = 25\ m/s$
Acceleration, $a = 2\ m/s^2$
From second equation of motion,
$v^2 - u^2 = 2as$
or $s = \frac{v^2-u^2}{2a}$
$= \dfrac{(25)^2 - (20)^2}{2 \times 2}\ m = 56.25\ m.$

Q3. D
Given,
Final velocity, $v = 0$
Acceleration, $a = -2\ m/s^2$
Initial velocity, $u =?$
$v = u + at$
$\Rightarrow 0 = u + -2 \times 20$
$\Rightarrow -u = -2 \times 20$
$\therefore u = 40\ m/s.$

Soln.

Q4. Here, initial speed, $u = \frac{90\ km}{hr}$

$= \left(90 \times \frac{5}{18}\right) m/s = 25\ m/s$

Acceleration, $a = -0.5\ m/s^2$

Train brought to rest, so final speed, $v = 0$

From third equation of motion,

$v^2 = u^2 + 2as$

$0 = (25)^2 - 2 \times 0.5 \times s$

$0 = 625 - s, \Rightarrow s = 625\ m$

The train will travel a distance of 625 m before it is brought to rest.

Q5. The motorboat starts from rest, so initial velocity, $u = 0$

Time taken $t = 8\ s$, acceleration, $a = 3\ ms^{-2}$

Displacement during the given time,

$s = ut + \frac{1}{2}at^2$

[$\because$ from second equation of motion]

$= 0 \times 8 + \frac{1}{2} \times 3 \times (8)^2 = \left(\frac{1}{2} \times 3 \times 64\right) m = 96\ m.$

Q6. From second equation of motion,

$s = ut + \frac{1}{2}at^2$

If the object starts from rest, i.e. $u = 0$

Then, $s = \frac{1}{2}at^2$

$s \propto t^2$, only if a =constant

So, the object moves with constant or uniform acceleration.

Q7. Here, $u = 5\ m/s$

$a = -10\ m/s^2$

[Negative sign is due to downward direction]

$v = 0$ (At maximum height velocity is zero)

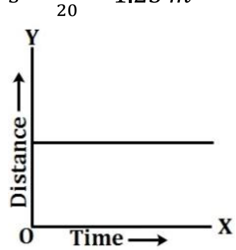
Height attained, s =?

Time taken, t =?

I. $v^2 = u^2 + 2as$

$0 = (5)^2 + 2 \times -10 \times s$

$s = \frac{25}{20} = 1.25\ m$



Height attained by the stone is 1.25 m

II. $v = u + at$

$0 = 5 + (-10)t$

$t = \frac{5}{10} = 0.5\ s$

The time taken by the stone to reach at the top will be 0.5 s

Q8. Here, $u = 20m/s, v = 0$

$s = 10cm = \frac{10}{100}m = \frac{1}{10}m$

Using $2as = v^2 - u^2$, we get

$2a \times \frac{1}{10} = (0)^2 - (20)^2$

Or $\frac{a}{5} = -400$

Or $a = (-400 \times 5)\ m/s^2 = -2000\ m/s^2$

i.e. Retardation caused by the target

$= 2000\ m/s^2$

Q9. According to the question,

	Stone 1	Stone 2
Initial velocity	u_1	u_2
Acceleration	$-g$	$-g$
Final velocity	$v_1 = 0$	$v_2 = 0$
Height	h_1	h_2

$v_1^2 = u_1^2 + 2gh_1$ and $v_2^2 = u_2^2 + 2gh_2$

Or $h_1 = \frac{-u_1^2}{-2g} = \frac{u_1^2}{2g}$ or $h_2 = \frac{-u_2^2}{-2g} = \frac{u_2^2}{2g}$

$h_1 : h_2 = \frac{\frac{u_1^2}{2g}}{\frac{u_2^2}{2g}} = u_1^2 : u_2^2$

ADVANCED QUESTIONS

Q1. I. Displacement of a particle at time t,

$s = ut + \frac{1}{2}at^2$

At time$(t-1)$, displacement of a particle,

$s' = u(t-1) + \frac{1}{2}a(t-1)^2$

$\therefore$Displacement in the last 1 s is $s - s'$

$= ut + \frac{1}{2}at^2 - \left[u(t-1) + \frac{1}{2}a(t-1)^2\right]$

$= ut + \frac{1}{2}at^2 - ut + u - \frac{1}{2}a(t-1)^2$

$= \frac{1}{2}at^2 + u - \frac{1}{2}at^2 - \frac{a}{2} + at$

$= u + \frac{a}{2}(2t-1)$

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II. At $u = 2\ m/s, a = 1\ m/s^2,\ t = 5\ s$

$$s = 2 + \frac{1}{2}(2 \times 5 - 1)\ m$$
$$= (2 + 4.5)\ m$$
$$= 6.5\ m.$$

Q2. I. Using $S_{nth} = u + \frac{a}{2}(2n - 1)$ we get (As it has been derived earlier)

Where S_{nth} represents the displacement of the particle in the n^{th} second

$$\therefore 3 = u + \frac{a}{2}(2 \times 1 - 1)$$
$$\Rightarrow 2u + a = 6 \qquad (1)$$

And $7 = u + \frac{a}{2}(2 \times 3 - 1)$

$$\Rightarrow 2u + 5a = 14 \qquad (2)$$

Subtracting (1) from (2), we get

$5a - a = 14 - 6$ or $4a = 8$

Or $a = 2m/s^2$

II. From (1), $u = \frac{6-a}{2}$

Or $u = \frac{6-2}{2}$

Or $u = \frac{4}{2}\ m/s$

$= 2\ m/s$

Q3. Here, $m = 0.006kg;\ u = 120m/s;$

$v = 0\ m/s$ (as the body is coming to rest)

$t = 0.01s$

Using $v = u + at$, we get

$$a = \frac{v - u}{t} = \frac{0 - 120}{0.01}$$
$$= \frac{-120 \times 100}{1}\ m/s^2$$
$$= -12000\ m/s^2$$

Using $2as = v^2 - u^2$, we get

$2(-12000)\ s = (0)^2 - (120)^2$

Or $-2 \times 12000 \times s = -120 \times 120$

$$= \frac{120 \times 120}{2 \times 12000}\ m$$
$$= \frac{6}{10} m$$
$$= 0.6m$$

Therefore the body covered a distance of $0.6\ m$ inside the target.

Q4. $v^2 = u^2 - 2gs \qquad (1)$

(In cases where a is equal to g, we can use the above equation)

At the highest point, h from the ground, $v = 0$

$\therefore 0 = u^2 - 2gh$ or $h = \frac{u^2}{2g} \qquad (2)$

When $s = \frac{h}{2}, v = 19.6m/s$

Therefore, from Equations (1) and (2),

$$(19.6\ m/s)^2 = u^2 - 2g\frac{u^2}{4g} = \frac{u^2}{2}$$
$$\therefore u^2 = 2(19.6\ m/s)^2$$
$$\therefore u = 27.71m/s$$

And $h = \dfrac{u^2}{2g}$

$$= \frac{(27.71\ m/s)^2}{(2 \times 9.8)\ m/s^2}$$
$$= 39.2\ m$$

The initial velocity of the ball is $27.71m/s$ and the maximum height reached by it is $39.2m$.

Q5. C

Assume the constant acceleration is $a\ m/s^2$ for $20\ s$.

Given: initial velocity, $(u) = 0$

Distance covered in first $10\ s = s_1$

Distance covered in last $10\ s = s_2$

To find: The relation between s_1 and s_2.

Using equation $s = ut + \frac{1}{2}at^2$

For the first case:

$s_1 = \frac{1}{2}a(10)^2 = 50\ a$

Speed at $t = 10\ s$ can be calculated by $v^2 = u^2 + 2as$

$v^2 = 0 + 2a\ (50\ a) = 100\ a^2$

$v = 10a$

Now $v = 10a$ will be the initial velocity for the last $10\ s$ of motion.

Using equation $s = ut + \frac{1}{2}at^2$

$s_2 = (10a) \times 10\ + \frac{1}{2}a \times (10)^2 = 150\ a$

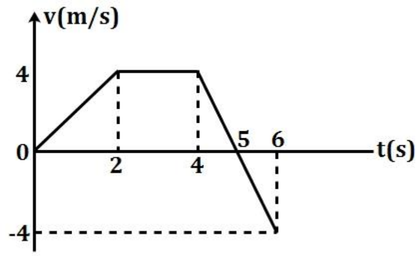
From the obtained values of s_1 and s_2

We get $s_2 = 3s_1$

Therefore, option C is correct.

ADVANCED PRACTICE PROBLEMS

Q1. From $t = 0$ to $t = 2s, a = +2m/s^2$



In the given scenario $u = 0$,
$\therefore v = u + at = 0 + 2t = 2t$
Or $v - t$ graph is a straight line passing through origin with slope $2m/s^2$
At the end of $2s$,
$v = 2 \times 2 = 4m/s$
From $t = 2$ to $4s, a = 0$. Hence, $v = 4m/s$ will remain constant
From $t = 4$ to $6s, a = -4m/s^2$.
$v = u + at$
In the time interval $t = 4s$ to $t = 6\ s$, as the acceleration is $-4\ m/s^2$
Substituting the value of a and u (which will be $4m/s$ for the next part of the motion) in the above equation we get $v = 4 - 4t$
After$1s$, the velocity is $v = 4 - 4 \times 1 = 0\ m/s$
Here we have taken the time as $1s$ and not $5s$ as the object obtains this motion only after $4s$.
At the end of $6s$ (or $t = 2s$) $v = (4 - 4 \times 2)m/s = -4m/s$. Corresponding $v - t$ graph is as shown alongside.

Q2. A.
Velocity has two components i.e. magnitude and direction. Change in any one component will change the velocity. Change in speed will definitely change the velocity, but change in velocity does not necessarily mean change in speed (it can be due to change in direction also). So option B can be ruled out.
Also if speed changes with the direction of motion being constant, velocity will change. Option D ruled out.
Acceleration does not necessarily mean change in speed. The best example can be of a uniform circular motion, where speed of the object remains constant but direction changes at every instant. Option C can also be ruled out.

Q3. C.
If we find the area under an acceleration-time graph till a particular time (for example till $20s$), we will get the instantaneous velocity of the object at that time (velocity at the 20^{th} second).
In the given figure, the area under the graph till $20s$ will be:
Area of trapezium + Area of rectangle
$\frac{1}{2}(3 + 5) \times 10\ m/s\ \ + (10 \times 5)\ m/s$
$\Rightarrow (40 + 50)\ m/s$

Q4. D.
All the three statements are true.
The object changes its direction of motion as the velocity changes its direction (from negative to positive).
As the slope of the graph is constant, the acceleration also remains constant.
The total displacement will be zero as the magnitude of positive displacement is equal to the magnitude negative displacement, so they will cancel out each other (as the directions are opposite).

Q5. A, D.
In the graphs given, only A and D show the motion of a particle with constant acceleration.
In B, the slope of the graph changes, so acceleration is not constant. In C, the velocity of the particle remains constant (slope of displacement-time graph is constant), as a result there is no acceleration.
In D the curve represents the arc of a circle, so the velocity changes at every instant (tangential to the $x - t$ curve). So there will be a constant acceleration as in case of a uniform circular motion.

Q6. Here, $u = 0; a = g$

Let distance fallen in time $t = h$

Therefore, using

$S = ut + \frac{1}{2}at^2$, we get $h = 0 \times t + \frac{1}{2}gt^2$

Or $h = \frac{1}{2}gt^2$ $(\because g = 10ms^{-2})$

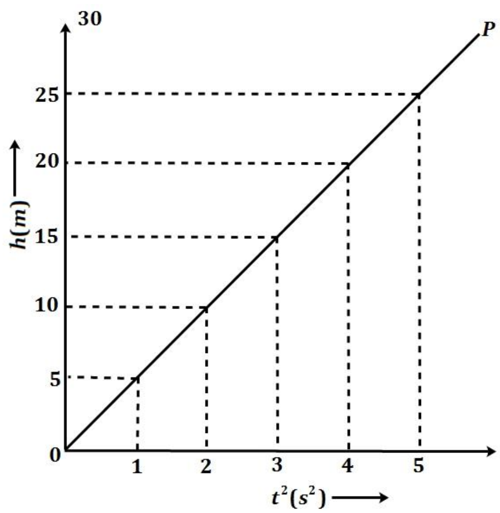
Or $h = \frac{1}{2} \times 10t^2$

Or $h = 5t^2$

Therefore, different values of t^2 and h are as follows:

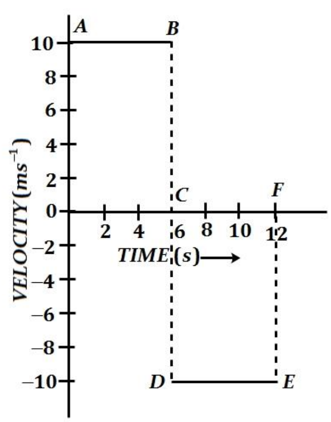
t^2 (in s^2)	0	1	2	3	4	5
h(in m)	0	5	10	15	20	25

Therefore, the graph (OP) between t^2 and h is a straight line as shown below



The value of g can be determined from this graph by taking double of the slope of the graph i.e. $g = 2 \times$ slope of $h - t^2$ graph $= 2 \times 5 = 10m\ s^{-2}$

Q7. The velocity – time graph of the ball is as follows:



Total distance travelled by the ball

$=$ Area $OABC +$ Area $CDEF = OA \times OC + CD \times CF = 10 \times 6 + 10 \times 6 = 120m$

Displacement of the ball = Area $OABC -$ Area $CDEF$

$= 60 - 60 = 0$

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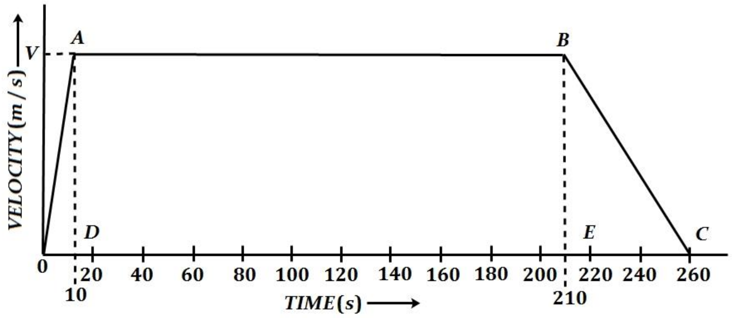
Q8. The velocity – time graph $(OABC)$ of the train is as shown ahead



I. Let maximum velocity reached by the train $= Vm/s$

Acceleration $= 2m/s^2$

Now, acceleration = slope of $OA = \frac{DA}{OD}$

$$\therefore 2 = \frac{V-0}{10-0} = \frac{V}{10}$$

$$\therefore V = (10 \times 2)m/s = 20m/s$$

II. Acceleration in the last $50s$ = slope of BC

$$= \frac{EB}{EC} = \frac{0-V}{260-210} = -\frac{V}{50} = -0.4\ m/s^2$$

i.e. Retardation in the last $50s = 0.4\ m/s^2$

III. Total distance travelled = area $OABC$

= Area of DOA + Area of rectangle $ABED$ + Area of BCE

$$= \frac{1}{2}OD \times AD + DE \times AD + \frac{1}{2}EC \times EB$$

$$= \tfrac{1}{2}(10-0) \times (V-0) + (210-10) \times (V-0) + \tfrac{1}{2}(260-210) \times (V-0)$$

$$= \frac{1}{2} \times 10 \times V + 200 \times V + \frac{1}{2} \times 50 \times V$$

$= 5V + 200V + 25V = 230V = 230 \times 20$ ($\because V = 20m/s$)

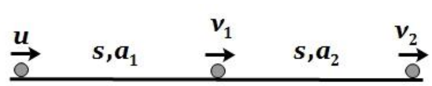
$= 4600m$

IV. Average velocity of the train

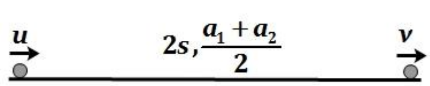
$$= \frac{\text{Total distance travelled}}{\text{Total time taken}} = \frac{4600m}{260s}$$

$$= \frac{230}{13} m/s = 17.69\ m/s$$

Q9. In the first case,



First case



Second case

$v_1^2 = u^2 + 2a_1 s$ (i)

$v_2^2 = v_1^2 + 2a_2 s$ (ii)

Adding Equations. (i) and (ii), we have

$v_2^2 = u^2 + 2\left(\frac{a_1+a_2}{2}\right)(2s)$ (iii)

In the second case

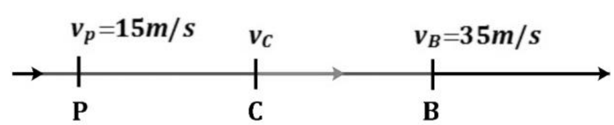
We are given that the acceleration is $\frac{a_1+a_2}{2}$ and the displacement is $2s$

Now substituting these values in the third equation of motion $v^2 = u^2 + 2as$, we get

$v^2 = u^2 + 2\left(\frac{a_1+a_2}{2}\right)(2s)$ (iv)

From Equations. (iii) and (iv), we can see that $v_2 = v$ Hence proved

Q10. Data: $v_P = 15m/s$ and $v_B = 35m/s$



Let $PC = CB = l$, where C is the midpoint of PB

$v^2 - u^2 = 2as$

$\therefore v_C^2 - v_P^2 = 2al$ and $v_{B^2} - v_{C^2} = 2al$

$\therefore v_C^2 - v_P^2 = v_B^2 - v_C^2$

$\therefore 2v_C^2 = v_B^2 + v_P^2 = (35\ m/s)^2 + (15\ m/s)^2$

$2v_C^2 = 1225 + 225 = 1450$

$\therefore v_C^2 = 725$

$\therefore v_C = \sqrt{725}\ m/s = 26.93\ m/s$

Q11. Data: $s_1 = 80m, s_2 = (80 + 100)m = 180m$, $u = 0\ m/s, a = 0.4m/s^2$

I. For the engine driver:

$$s_1 = ut_1 + \frac{1}{2}at_1^2$$

$$\therefore 80m = 0 + \frac{1}{2}(0.4\ m/s^2)t_1^2$$

$$\therefore t_1^2 = \frac{160}{0.4}\ s^2 = 400s^2$$

$\therefore$ The time taken by the engine driver to pass the signal, $t_1 = 20s$

20 seconds after starting from rest, the speed of the train is

$v_1 = u_1 + at_1 = 0 + \left(0.4\frac{m}{s^2}\right)(20\ s)$

$= 8\ m/s$

II. For the guard:

$s_2 = ut_2 + \frac{1}{2}at_2^2$

$\therefore 180m = 0 + \frac{1}{2}(0.4\ m/s^2)t_2^2$

$\therefore t_2^2 = \frac{360}{0.4}\ s^2 = 900\ s^2$

$\therefore$ The time taken by the guard to pass the signal, $t_2 = 30s$

30 seconds after starting from rest, the speed of the train is

$v_2 = u_2 + at_2 = 0 + (0.4\ m/s^2)(30s)$

$= 12\ m/s$

Q12. I. The particle moved with uniform velocity of $20m/s$ in the region AB from AB from $t = 1s$ to $t = 2s$, and $10m/s$ in the region CD from $t = 3s$ to $t = 4s$

II. The particle moved with uniform acceleration in the regions OA and BC

In the region OA,

$a = \frac{\Delta v}{\Delta t} = \frac{20m/s\ - 0\ m/s}{1s - 0s} = 20m/s^2$ (positive)

In the region BC,

$a = \frac{\Delta v}{\Delta t} = \frac{10m/s\ -\ 20m/s}{3s - 2s} = \frac{-10}{1}\ m/s^2 = -10m/s^2$ (negative)

i.e., there was uniform deceleration

III. The distance traversed by the particle 4 seconds = Area under the curve $OABCD$ = Area of ΔOAH + Area of rect. $ABGH$ + Area of trapezium $BCFG$ + Area of rect. $CDEF$

$= \frac{1}{2} \times 1 \times 20 + 1 \times 20 + \frac{1}{2} \times 1 \times (10 + 20) + 1 \times 10$

$= (10 + 20 + 15 + 10)\ m = 55m$

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