Indian National Astronomy Olympiad – 2017

Question Paper | INAO – 2017

Roll Number: ☐☐☐☐ - ☐☐☐☐ - ☐☐☐☐ | Date: 28th January 2017

Duration: **Three Hours** | Maximum Marks: 100

Please Note:

- Please write your roll number on top of this page in the space provided.
- Before starting, please ensure that you have received a copy of the question paper containing total 3 pages (6 sides).
- There are total 7 questions. Maximum marks are indicated in front of each sub-question.
- For all questions, the process involved in arriving at the solution is more important than the answer itself. Valid assumptions / approximations are perfectly acceptable. Please write your method clearly, explicitly stating all reasoning.
- Blank spaces are provided in the question paper for the rough work. No rough work should be done on the answer-sheet.
- Non-programmable scientific calculators are allowed.
- **The answer-sheet must be returned to the invigilator**. You can take this question booklet back with you.
- Please be advised that tentative dates for the next stage are as follows:
  - Orientation Cum Selection Camp (Senior): 22nd April to 8th May 2017. This will be held at HBCSE, Mumbai.
  - IAO selection camp (junior) will be held at Banglore and dates will be announced by NCSM later.
  - Attending the camp for the entire duration is mandatory for all participants.

Useful Physical Constants

Mass of the Sun	$M_\odot$	$\approx$	1.989×10^{30} kg
Radius of the Sun	$R_\odot$	$\approx$	6.955×10^{8} m
1 parsec (pc)		$=$	3.086×10^{16} m
Solar Luminosity	$L_\odot$	$\approx$	3.826×10^{26} W
Solar Constant (at Earth)	S	$=$	1366 W m^{-2}
Gravitational Constant	G	$\approx$	6.674×10^{-11} N m^{2} kg^{-2}
Stefan's Constant	σ	$=$	5.670×10^{-8} W m^{-2}K^{-4}

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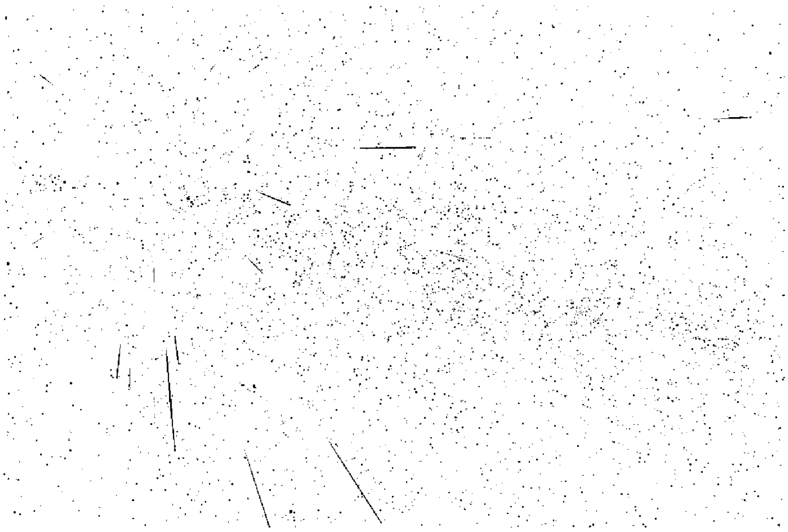
1. (15 marks) In each of the subquestions below, some astronomical phenomenon is shown through one or more images. Identify the phenomenon and describe it in 3-4 sentences.

   (a) Image below is a long time exposure of a certain part of the sky. Explain what you see in the image.



Solution:
Meteor shower. All the meteors appear to originate from the same point in the sky when the trajectories are extrapolated backwards.

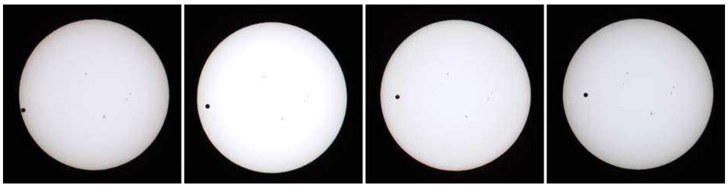
   (b) Explain what you see in the image below.

Solution:
This phenomenon is called the Occultation of Saturn, where the Moon passes in front of Saturn. The occultation starts from the dark edge of the Moon.

   (c) Image below includes multiple shots taken on same background at different times. Explain what you see in the image.

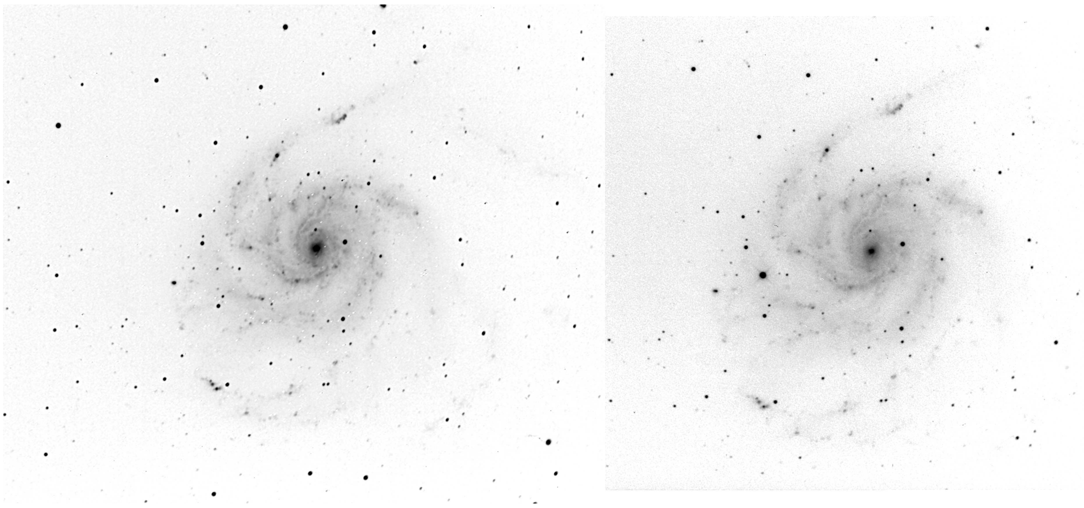
Solution:
Retrograde motion of a planet. The apparent motion of a planet in direction opposite to that of other bodies (background stars), as observed from a Earth is called retrograde motion.

(d) Each image below is showing same object but they are taken at different times. Explain what you see in the image.



Solution:
Transit of an inner planet over the solar disc. The disc of the planet is seen as a dark spot.

(e) Two images below are showing same object but they are taken at different times. Explain what you see in the image.

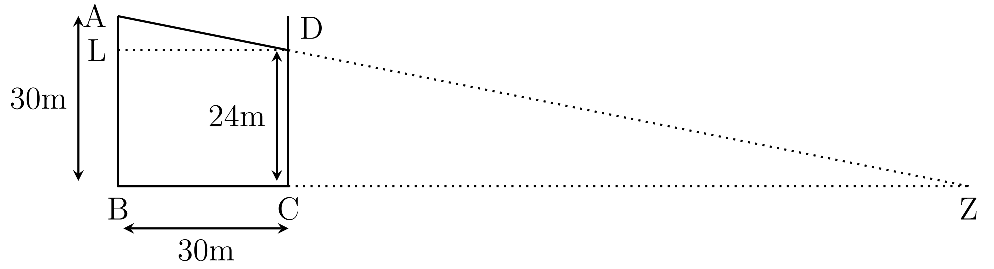


Solution:
Supernova. Here a very bright star has suddenly appeared in the galaxy as seen in the picture on the right. This can only be a supernova.

2. On 22$^{\text{nd}}$ December, part of the shadow of a 30 m tall building falls on another building next to it. The separation between the two buildings is 30 m and height of the shadow falling on the adjacent building is 24 m.

   (a) (2 marks) How long would the shadow be (on the ground) in the absence of this adjacent building?

Solution:

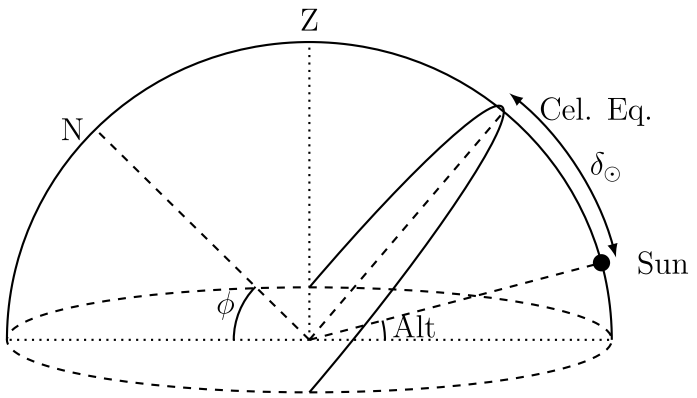


From the figure,
In △ ALD, $\dfrac{AL}{AD} = \dfrac{1}{5}$

Since, △ ABZ is similar to △ ALD,
l(BZ) = 150 m.
Hence, the length of the shadow in absence of the building is 150 m.

   (b) (5 marks) If this shadow is noticed at the local noon, what is the latitude of the place?

Solution:



From the pervious part we know that, tan ∠ ADL $= \dfrac{1}{5} = 11.31°$
On 22$^{\text{nd}}$ December Sun is $\delta_\odot = -23.5°$ below the celestial equator,
From the diagram, we can obtain the following relation between the declination and altitude of Sun and latitude of the place,
latitude = 90°−(declination + altitude) 3 M
latitude = $90° - (23.5 + 11.31)$
Therefore the latitude of the place is 55.2° 2 M

3. A star, at a distance of 1 pc from the Earth, becomes a supernova and reaches a maximum luminosity of $10^{11} L_\odot$.

(a) (3 marks) What will be the maximum flux, F_{sup}, of this supernova at Earth?

Solution:

The flux would be given by,

$$F_{\text{sup}} = \frac{L_{\text{sup}}}{4\pi d_{\text{sup}}^2} = \frac{10^{11} L_\odot}{4\pi (1\,\text{pc})^2}$$ 2 M

$$= \frac{10^{11} \times 3.826 \times 10^{26}}{4\pi (3.086 \times 10^{16})^2}$$

$$F_{\text{sup}} = 3197\,\text{W/m}^2$$ 1 M

(b) (3 marks) By what factor, R, will the total light flux on the Earth's surface increase over that from the Sun alone?

Solution:

Comparing it with the solar constant,

$$\text{Ratio} = \frac{F_{\text{sup}} + S}{S}$$ 2 M

$$= \frac{3197 + 1366}{1366}$$

$$\text{Ratio} = 3.340$$ 1 M

(c) (4 marks) What will be the temperature, T, of the Earth when it reaches a new thermal equilibrium after the supernova? Assume the Earth to be a perfect black body and that it reaches a new equilibrium within a few days. Observations show that the average temperature of the Earth during the last century was 287 K.

Solution:

By Stefan's Law,

$$\frac{T^4}{T_0^4} = \frac{F_{\text{sup}} + S}{S} = 3.34$$ 2 M

$$\therefore T = T_0 \times 3.34^{0.25} \simeq 287 \times 1.35$$

$$T \simeq 388\,\text{K} \simeq 115\,^\circ\text{C}$$ 2 M

(d) (5 marks) Human life can survive on Earth only if the equillibrium temperature remains below 333 K. Find the minimum distance, d_{sup}, in pc for this supernova, if the life on the Earth should survive?

Solution:

using relation in the previous part,

$$\frac{F_1 + S}{S} = \frac{T_1^4}{T_0^4}$$

$$\therefore F_1 = S\left(\frac{T_1^4}{T_0^4} - 1\right) = 1366 \times \left(\frac{333^4}{287^4} - 1\right)$$ 2 M

$$F_1 \simeq 1110\,\mathrm{W/m^2}$$

$$F_1 = \frac{L_s}{4\pi d_{\mathrm{sup}}^2}$$ 0.5 M

$$\therefore d_{\mathrm{sup}} = \sqrt{\frac{L_s}{4\pi F_1}}$$ 2 M

$$= \sqrt{\frac{1 \times 10^{11} \times 3.826 \times 10^{26}}{4\pi \times 1110}}$$

$$= 5.24 \times 10^{16}\,\mathrm{m}$$

$$d_{\mathrm{sup}} \geq 1.70\,\mathrm{pc}$$ 0.5 M

(e) (5 marks) Assume that the number density of stars in the Milky Way is $0.14\,\mathrm{pc}^{-3}$. There are 10^{11} stars uniformly distributed across the galaxy. Also assume that there is one supernova every 30 years and all of them have same luminosity. Find the probablity, P, of a supernova causing extinction on Earth in total life span of the Sun.

Solution:
Given this distribution of stars, the number of stars which can end in life threatening supernova are,

$$N_{st} = \rho_n \times \frac{4}{3}\pi d^3 = 0.14 \times \frac{4}{3}\pi 1.7^3$$ 2 M

$$N_{st} \approx 2.88$$ 0.5 M

$$p \approx \frac{N_{\mathrm{st}}}{N_{\mathrm{total}}} \times \frac{T_\odot}{T_{\mathrm{supernova}}}$$ 2 M

$$p \approx \frac{2.88}{10^{11}} \times \frac{10^{10}}{30}$$

$$p \approx 0.0096$$ 0.5 M

4. Chinmay was standing outside Talegaon bus station one morning. He noted down timing of each bus as it left the bus station. The table below lists the bus timings noted by him. His friend Nikhil told him later that buses from this bus station typically go to three destinations. A bus leaves every x minutes for Pune, a bus leaves every y minutes for Nashik and a bus leaves every z minutes for Mumbai ($x < y < z$). During these three hours, one bus also leaves for Nagpur.

	Time		**Time**		**Time**		**Time**		**Time**
1	07:01	12	07:38	23	08:13	34	08:48	45	09:25
2	07:03	13	07:41	24	08:20	35	08:53	46	09:30
3	07:09	14	07:43	25	08:21	36	08:53	47	09:33
4	07:10	15	07:49	26	08:23	37	09:01	48	09:33
5	07:13	16	07:52	27	08:29	38	09:02	49	09:41
6	07:17	17	07:53	28	08:33	39	09:03	50	09:43
7	07:23	18	07:57	29	08:34	40	09:09	51	09:44
8	07:24	19	08:03	30	08:37	41	09:13	52	09:49
9	07:25	20	08:05	31	08:43	42	09:16	53	09:53
10	07:33	21	08:06	32	08:45	43	09:17	54	09:57
11	07:33	22	08:13	33	08:46	44	09:23	55	09:58

(a) (6 marks) Find values of x, y and z. Explain your method briefly.

Solution:
From the data we notice,

- There is a 8 minute gap between observation 9 and 10. This means x, which is smallest of the three, cannot be less than 8 minutes.
- Let us temporarily assume that the Nagpur bus is not among the first four buses in the morning. Then, out of first four buses, at least two should be heading for Pune. Thus, possible values for x are 2 minute, 8 minutes or 9 minutes.
- The first value (2 minutes) is ruled out due to the first point. if we take x to be 9 minutes, then we are assuming bus at 07:01 and 07:10 are headed for Pune. In that case, there should be a bus for Pune at 07:19. But there is no such bus. So 9 minutes is also ruled out.
- If we take $x = 8$ minutes, we see the pattern matching with the bus starting at 07:01. We fix the value of x and eliminate all buses in that series.
- For the remaining buses, again we apply same rule. Out of the buses leaving at 07:03, 07:10 and 07:13 at least two should be headed for Nashik. As y is greater than x, only logical choice is $y = 10$ minutes.
- After eliminating second sequence, we find out $z = 14$ minutes.

Marking scheme: 2 marks for correct value of x, y and z with logic.

(b) (2 marks) When did the bus going to Nagpur leave the bus station?

Solution:
After eliminating all the three series, the only bus which remains is the bus that leaves the station at 08:46. Hence, this is the bus leaving for Nagpur. 2 M

5. Dhruv is transported to an alternate universe where laws of physics are different from what they are in our world. Specifically, Snell's law for refraction in this alternate universe takes the form

$$n_1 \tan i = n_2 \tan r$$

where n_1, n_2, i and r have their usual meanings. Refractive indices of different materials remain the same in this universe. Use this law to answer the following questions:

(a) (2 marks) A ray of light is incident on surface of water at 30°. What will be the angle of refraction in this case?

Solution:

The refractive index of air is 1 and that of water is 1.33,

$\tan r = \dfrac{\tan 30°}{1.33} \Rightarrow r = 23.46° \approx 23°$ 2 M

(b) (5 marks) What will be condition for mirage in this universe?

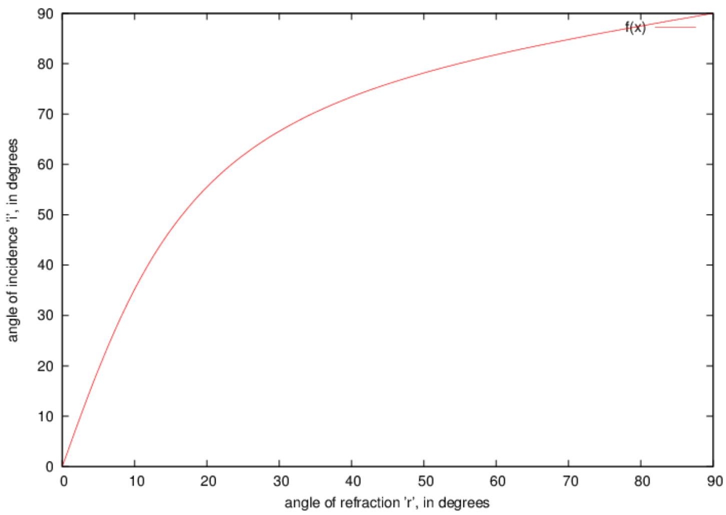
Solution:

Mirage requires total internal reflection, i.e., $r = 90°$. 2 M

For this, we require $i = 90°$, i.e., grazing incidence. For grazing incidence, there is no refraction. Therefore, TIR, and hence mirage, is not possible in this universe. 3 M

(c) (5 marks) Make a schematic plot of i vs. r, if medium 1 is air and medium 2 has refractive index of 4.00.

Solution:



Marking Scheme:

- Scale 1 M
- Axis labels 1 M
- Curvature 2 M

- Ranges 1 M

(d) (5 marks) Dhruv has vertically placed a cylindrical tube 10 cm long with a light collecting device at the bottom. The tube is filled with medium 2. What should be the radius of the tube if light from any star with an altitude of 45° and more were to be incident on this device?

Solution:
From the previous part, $n_2 = 4$ and $n_1 = 1$,
Length of the tube filled with medium 2 is 10 cm
For a star at an altitude of 45°, the angle of incidence i will be 45°. Hence,

$$\tan r = \frac{n_1}{n_2} \times \tan i$$

$$\tan r = 0.25$$

$$\text{Also,} \tan r = \frac{\text{diameter of the tube}}{10\,\text{cm}}$$

Hence the radius of the tube should be 1.25 cm. 5 M

We use diameter of the tube because we are merely considering a light collecting device at the bottom of the tube and not an imaging device. **Marking Scheme : 3 Marks if students work it out for radius and not for diameter.**

(e) (3 marks) Do you agree with the statement below? Justify your answer.
"Exactly 50% of the sky above horizon can be viewed through this device at any given time."

Solution:
No. For a unit sphere, solid angle, Ω, subtended by a cone whose cross-section subtends an angle 2θ is given by,
$\Omega = 2\pi(1 - \cos\theta)$
For a hemisphere, solid angle subtended is 2π steradians, and that subtended by a cone whose cross-section subtends an angle 90°, is $\pi(2 - \sqrt{2})$steradians.
Marking scheme : With explanation 3 Marks

6. Recently Mr. Jawkar visited his friend Mr. Anthony's house. Although he was visiting them after many years, he had heard that besides the two married adults in the house there are two children of different ages. But he did not know their genders. When he knocked at the door a boy answered.

   (a) (4 marks) What is the probability that the other child is a girl?

Solution:
Here the total sample space will be as follows;

Sr. No.	Frist Child (Elder)	Second Child (Younger)	One who goes to attend the door
1	B_1	B_2	B_1
2	B_1	B_2	B_2
3	B	G	B
4	B	G	G
5	B	B	B
6	B	B	G
7	G_1	G_2	G_1
8	G_1	G_2	G_2

The condition in the question is, that a boy opens the door, therefore we are left with only four of the above eight possibilities, hence the total sample space is reduced to {1,2,3,5}. Since, the probability for the other child being a girl is to be calculated, the favourable outcomes are {3,5} i.e. younger boy **&** elder girl **OR** elder boy **&** younger girl. Therefore, the probability that the other child is a girl is $\frac{2}{4} = \frac{1}{2}$.

Note:

- Simply stating gender of other child is independent of who opens the door, will not get full credit.
- Partial credit will also be given for $P = \frac{2}{3}$.

   (b) (2 marks) If the boy says, "I am the elder one", what is the probability that the other child is a girl?

Solution:
In this case the sample space is reduced to {1,3} because the boy mentions he is the elder one. As the probability for the other child being a girl is to be calculated, the favourable outcome is just {3} i.e. elder boy **&** younger girl. Therefore, the probability that the other child being a girl is $\frac{1}{2}$.

   (c) (6 marks) Mr. Jawkar was carrying with him flags of 40 countries and 40 labels of the corresponding country names. He wanted Mr. Anthony's family to help him correctly assign labels to the country flags. He equally divided the flags and labels (at random) amongst the 4 family members of Mr. Anthony. He told them to put labels on the flags assigned to each of them but they were not allowed to exchange flags or labels with each other. What is the probability that all 40 flags get correct labels?

Solution:

Let A_1, A_2, A_3 and A_4 be the members of the Mr. Anthony's family.
Probability that A_1 picks up the correct label for the first flag in his slot is, $\frac{10}{40}$, and for the next flag the probability is $\frac{9}{39}$ and so on so forth upto $\frac{1}{31}$.

Probability that A_2 picks up the correct label for the first flag in his slot is, $\frac{10}{30}$, for next flag the probability is $\frac{9}{29}$ and so on so forth upto $\frac{1}{21}$.

Similarly, for A_3 it is $\frac{10}{20}$ and $\frac{10}{19}$ and so on so forth upto $\frac{1}{11}$, and for A_4 is from $\frac{10}{10}$ upto $\frac{1}{1}$.

Therefore, the total probability for A_1 getting all the flags and labels correct is,
$P_{A_1} = \frac{10}{40} \times \frac{9}{39} \times \frac{8}{38} \times \frac{7}{37} \times \frac{6}{36} \times \frac{5}{35} \times \frac{4}{34} \times \frac{3}{33} \times \frac{2}{32} \times \frac{1}{31}$

and the total probability for A_2 getting all the flags and labels correct is,
$P_{A_2} = \frac{10}{30} \times \frac{9}{29} \times \frac{8}{28} \times \frac{7}{27} \times \frac{6}{26} \times \frac{5}{25} \times \frac{4}{24} \times \frac{3}{23} \times \frac{2}{22} \times \frac{1}{21}$

similarly, for A_3 getting all the flags and labels correct is,
$P_{A_3} = \frac{10}{20} \times \frac{9}{19} \times \frac{8}{18} \times \frac{7}{17} \times \frac{6}{16} \times \frac{5}{15} \times \frac{4}{14} \times \frac{3}{13} \times \frac{2}{12} \times \frac{1}{11}$

If all the above combinations are correct probability of A_4 getting all the flags and labels correct is one,
$P_{A_4} = 1$
Hence, the probability that all 40 flags get correct labels is,

$$P = P_{A_1} \times P_{A_2} \times P_{A_3} \times P_{A_4} = \frac{(10!)^3}{40 \times 39 \times .... \times 11} \times 1 = \frac{(10!)^4}{40!}$$

(d) (3 marks) What is the probability that exactly one flag gets wrongly labelled?

Solution:

Probability is **zero**. If 39 labels are correctly assigned then the last flag is bound to get correct label.

7. (15 marks) Keyuri is holding a spherical balloon with a radius of 40.00 cm. Two ants were standing next to each other on the surface of this balloon. One of the ants starts walking away from the other ant with a linear speed of $2.500\,\mathrm{mm\,s^{-1}}$ as measured by Keyuri. At the same time, Keyuri starts pumping air inside the balloon at the rate of $0.3142\,\mathrm{cm^3/s}$. This slow pumping does not change pressure or temperature of air inside the balloon. What will be the separation between the two ants after 6.400 s? Ignore the sizes of the ants.

Solution:

Let r be the radius of the balloon, let v be the speed of the moving ant and t be the time. Let the two ants subtend angle θ at the centre of balloon. As the separation between ants is very small as compared to the circumference of a great circle of the sphere, we may approximate the distance between the ants along the surface of the sphere as $d = r\theta$. 5 M

If the initial radius of the balloon is r_0, and the rate of increase of volume of the balloon is α then, at a time $t(>0)$, the volume, V of the balloon is,

$$V = \frac{4\pi}{3}r^3 = \frac{4\pi}{3}{r_0}^3 + \alpha t$$
$$r^3 = {r_0}^3 + \frac{3\alpha t}{4\pi}$$
$$r = \sqrt[3]{{r_0}^3 + \frac{3\alpha t}{4\pi}}$$

As a first order approximation, the separation between ants will be 5 M

$$\begin{aligned} d &= \theta r \\ &= \frac{vt}{r_0}\sqrt[3]{{r_0}^3 + \frac{3\alpha t}{4\pi}} = vt\sqrt[3]{1 + \frac{3\alpha t}{4\pi {r_0}^3}} \\ &= 0.25 \times 6.4\sqrt[3]{1 + \frac{3 \times 0.3142 \times 6.4}{4\pi \times 40^3}} \\ &= 1.6\sqrt[3]{1 + 0.0000075} \\ &= 16.000\,04\,\mathrm{mm} \end{aligned}$$

3 Marks for calculations and 2 Marks for final answer. As the values in the question are given only upto 4 significant digits the answer maybe rounded off to 16.00 mm. However, no points for rounded off answer will be given if students have not checked for the effect of expansion or have rounded off the answers too early.

Space for rough work.

Notes for IAOSP candidates

- **Blackbody -**
  An ideal blackbody absorbs all thermal radiation incident on it. A blackbody at thermal equilibrium,
  1. emits as much energy as it absorbs,
  2. emits in all directions equally.
- **Stefan-Boltzmann Law -**
  The thermal energy radiated by a blackbody per second per unit area is proportional to the fourth power of the absolute temperature and is given by
  $$\frac{L}{A} = \sigma T^4 \quad \mathrm{Jm^{-2}\ s^{-1}}$$
  where L is luminosity, A is area and σ is Stefan's constant.
- **Probability -**
  Probability of a certain result in any experiment is given by,
  $$\text{Probability} = \frac{\text{total number of ways of getting that particular result}}{\text{total number of possible results}}$$

Space for rough work.