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Senior School Certificate Examination

March 2016

Marking Scheme — Mathematics 65/1/C, 65/2/C, 65/3/C

General Instructions:

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggested answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage

2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. In question (s) on differential equations, constant of integration has to be written.

5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.

6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.

7. Separate Marking Scheme for all the three sets has been given.

8. As per orders of the Hon'ble Supreme Court. The candidates would now be permitted to obtain photocopy of the Answer book on request on payment of the prescribed fee. All examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

QUESTION PAPER CODE 65/1/C

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $(x + 3)2x - (-2)(-3x) = 8$ $\frac{1}{2}$

$x = 2$ $\frac{1}{2}$

2. $\begin{pmatrix} 2 & 5 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -1 & -1 \end{pmatrix}$ $\frac{1}{2} + \frac{1}{2}$

3. $\left.\begin{array}{r} \text{No. of possible matrices} = 3^4 \\ \text{or} \quad 81 \end{array}\right\}$ 1

4. $\dfrac{2(2\vec{a} + 3\vec{b}) + 1(3\vec{a} - 2\vec{b})}{2+1}$ $\frac{1}{2}$

$= \frac{7}{3}\vec{a} + \frac{4}{3}\vec{b}$ (or enternal division may also be considered) $\frac{1}{2}$

5. 2 1

6. $\frac{x}{3} + \frac{y}{-4} + \frac{z}{2} = 1$ $\frac{1}{2}$

$\Rightarrow \vec{r}\cdot(4\hat{i} - 3\hat{j} + 6\hat{k}) = 12$ or $\vec{r}\cdot\left(\frac{\hat{i}}{3} - \frac{\hat{j}}{4} + \frac{\hat{k}}{2}\right) = 1$ $\frac{1}{2}$

SECTION B

7. Equation of line through A(3, 4, 1) and B(5, 1, 6)

$\frac{x-3}{2} = \frac{y-4}{-3} = \frac{z-1}{5} = k(\text{say})$ 1

General point on the line:

$x = 2k + 3,\ y = -3k + 4,\ z = 5k + 1$ $\frac{1}{2}$

line crosses xz plane i.e. $y = 0$ if $-3k + 4 = 0$

$\therefore \quad k = \frac{4}{3}$ 1

Co-ordinate of required point $\left(\frac{17}{3}, 0, \frac{23}{3}\right)$ $\frac{1}{2}$

Angle, which line makes with xz plane:

$\sin\theta = \left|\dfrac{2(0) + (-3)(1) + 5(0)}{\sqrt{4+9+25}\sqrt{1}}\right| = \dfrac{3}{\sqrt{38}} \Rightarrow \theta = \sin^{-1}\left(\dfrac{3}{\sqrt{38}}\right)$ 1

8. let $\vec{d}_1$ & $\vec{d}_2$ be the two diagonal vectors:

$\therefore \quad \vec{d}_1 = 4\hat{i} - 2\hat{j} - 2\hat{k}, \ \vec{d}_2 = -6\hat{j} - 8\hat{k}$ $\frac{1}{2} + \frac{1}{2}$

or $\quad \vec{d}_2 = 6\hat{j} + 8\hat{k}$

Unit vectors parallel to the diagonals are:

$\hat{d}_1 = \frac{2}{\sqrt{6}}\hat{i} - \frac{1}{\sqrt{6}}\hat{j} - \frac{1}{\sqrt{6}}\hat{k}$ $\frac{1}{2}$

$\hat{d}_2 = -\frac{3}{5}\hat{j} - \frac{4}{5}\hat{k} \quad \left(\text{or } \hat{d}_2 = \frac{3}{5}\hat{j} + \frac{4}{5}\hat{k}\right)$ $\frac{1}{2}$

$\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -2 & -2 \\ 0 & -6 & -8 \end{vmatrix} = 4\hat{i} + 32\hat{j} - 24\hat{k}$ 1

Area of parallelogram = $\frac{1}{2}\left|\vec{d}_1 \times \vec{d}_2\right| = \sqrt{404}$ or $2\sqrt{101}$ sq. units 1

9. let X = Amount he wins then x = ₹ 5, 4, 3, – 3 1

P = Probability of getting a no. >4 = $\frac{1}{3}$, q = $1 - p = \frac{2}{3}$ $\frac{1}{2}$

X:	5	4	3	–3
P(x)	$\frac{1}{3}$	$\frac{2}{3}\cdot\frac{1}{3} = \frac{2}{9}$	$\left(\frac{2}{3}\right)^2\cdot\frac{1}{3} = \frac{4}{27}$	$\left(\frac{2}{3}\right)^3 = \frac{8}{27}$

2

Expected amount he wins $= \Sigma XP(X) = \frac{5}{3} + \frac{8}{9} + \frac{12}{27} - \frac{24}{27}$

$= ₹\,\frac{19}{9}$ or ₹ $2\frac{1}{9}$ $\frac{1}{2}$

OR

E_1 = Event that all balls are white,

E_2 = Event that 3 balls are white and 1 ball is non white

E_3 = Event that 2 balls are white and 2 balls are non-white

A = Event that 2 balls drawn without replacement are white 1

$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$ $\frac{1}{2}$

$P(A/E_1) = 1,\ P(A/E_2) = \frac{3}{4}\cdot\frac{2}{3} = \frac{1}{2},\ P(A/E_3) = \frac{2}{4}\cdot\frac{1}{3} = \frac{1}{6}$ $1\frac{1}{2}$

$P(E_1/A) = \frac{1.\frac{1}{3}}{1\cdot\frac{1}{3} + \frac{1}{3}\cdot\frac{1}{2} + \frac{1}{3}\cdot\frac{1}{6}} = \frac{3}{5}$ 1

10. let $y = u + v$, $u = x^{\sin x}$, $v = (\sin x)^{\cos x}$

$\log u = \sin x.\log x \Rightarrow \frac{du}{dx} = x^{\sin x} \cdot \left\{\cos x.\log x + \frac{\sin x}{x}\right\}$ $\quad \frac{1}{2}+1$

$\log v = \cos x.\log (\sin x) \Rightarrow \frac{dv}{dx} = (\sin x)^{\cos x} \cdot \{\cos x \cdot \cot x - \sin x \cdot \log(\sin x)\}$ $\quad \frac{1}{2}+1$

$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} = x^{\sin x} \cdot \left\{\cos x \cdot \log x + \frac{\sin x}{x}\right\} + (\sin x)^{\cos x}\{\cos x \cdot \cot x - \sin x \cdot \log(\sin x)\}$ $\quad \frac{1}{2}+\frac{1}{2}$

OR

$\frac{dy}{dx} = \frac{-2\sin(\log x)}{x} + \frac{3\cos(\log x)}{x}$ $\quad 1$

$\Rightarrow \quad x\frac{dy}{dx} = -2\sin(\log x) + 3\cos(\log x)$, differentiate w.r.t 'x' $\quad \frac{1}{2}$

$\Rightarrow \quad x\frac{d^2y}{dx^2} + \frac{dy}{dx} = \frac{-2\cos(\log x)}{x} - \frac{3\sin(\log x)}{x}$ $\quad 2$

$\Rightarrow \quad x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -y \Rightarrow x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ $\quad \frac{1}{2}$

11. $\frac{dx}{dt} = 2a\cos 2t\,(1 + \cos 2t) - 2a\sin 2t\cdot\sin 2t$ $\quad 1\frac{1}{2}$

$\frac{dy}{dt} = -2b\sin 2t\,(1 - \cos 2t) + 2b\cos 2t\cdot\sin 2t$ $\quad 1$

$\left.\frac{dy}{dx}\right]_{t=\frac{\pi}{4}} = \left.\frac{2b\cos 2t\cdot\sin 2t - 2b\sin 2t\,(1-\cos 2t)}{2a\cos 2t\,(1+\cos 2t) - 2a\sin 2t\cdot\sin 2t}\right]_{t=\frac{\pi}{4}} = \frac{b}{a}$ $\quad \frac{1}{2}+1$

12. $y^2 = ax^3 + b \Rightarrow 2y\frac{dy}{dx} = 3ax^2 \therefore \frac{dy}{dx} = \frac{3a}{2}\frac{x^2}{y}$ $\quad 1$

Slope of tangent at (2, 3) $= \left.\frac{dy}{dx}\right]_{(2,3)} = \frac{3a}{2}\cdot\frac{4}{3} = 2a$ $\quad 1$

Comparing with slope of tangent $y = 4x - 5$, we get, $2a = 4 \therefore \boxed{a = 2}$ $\quad 1$

Also (2, 3) lies on the curve $\therefore 9 = 8a + b$, put $a = 2$, we get $b = -7$ $\quad 1$

13. Let $x^2 = t \therefore \frac{x^2}{x^4 + x^2 - 2} = \frac{x^2}{(x^2-1)(x^2+2)} = \frac{t}{(t-1)(t+2)} = \frac{A}{t-1} + \frac{B}{t+2}$ $\quad 1$

Solving for A and B to get, $A = \frac{1}{3}, B = \frac{2}{3}$ $\quad 1$

$\int \frac{x^2}{x^4+x^2-2}dx = \frac{1}{3}\int\frac{1}{x^2-1}dx + \frac{2}{3}\int\frac{1}{x^2+2}dx = \frac{1}{6}\log\left|\frac{x-1}{x+1}\right| + \frac{\sqrt{2}}{3}\tan^{-1}\frac{x}{\sqrt{2}} + C$ $\quad 1 + 1$

14. Let $I = \int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x} dx$, Also $I = \int_0^{\pi/2} \frac{\sin^2\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)} dx = \int_0^{\pi/2} \frac{\cos^2 x}{\cos x + \sin x} dx$ 1

Adding to get, $2I = \int_0^{\pi/2} \frac{1}{\sin x + \cos x} dx = \frac{1}{\sqrt{2}} \int_0^{\pi/2} \frac{1}{\cos(x - \pi/4)} dx$ $\frac{1}{2} + 1$

$\Rightarrow 2I = \frac{1}{\sqrt{2}} \int_0^{\pi/2} \sec(x - \pi/4)\, dx = \frac{1}{\sqrt{2}} \log\left|\sec\left(x - \frac{\pi}{4}\right) + \tan\left(x - \frac{\pi}{4}\right)\right|_0^{\pi/2}$ 1

$\Rightarrow 2I = \frac{1}{\sqrt{2}}\left\{\log\left|\sqrt{2}+1\right| - \log\left|\sqrt{2}-1\right|\right\}$

$\Rightarrow I = \frac{1}{2\sqrt{2}}\left\{\log\left|\sqrt{2}+1\right| - \log\left|\sqrt{2}-1\right|\right.$ or $\frac{1}{2\sqrt{2}} \log\left|\frac{\sqrt{2}+1}{\sqrt{2}-1}\right|$ $\frac{1}{2}$

OR

$\int_0^{3/2} |x \cos \pi x|\, dx = \int_0^{1/2} x \cos \pi x\, dx - \int_{1/2}^{3/2} x \cos \pi x\, dx$ $1\frac{1}{2}$

$= \left\{\frac{x \sin \pi x}{\pi} + \frac{\cos \pi x}{\pi^2}\right\}_0^{1/2} - \left\{\frac{x \sin \pi x}{\pi} + \frac{\cos \pi x}{\pi^2}\right\}_{1/2}^{3/2}$ $1\frac{1}{2}$

$= \frac{1}{2\pi} - \frac{1}{\pi^2} - \left(-\frac{3}{2\pi} - \frac{1}{2\pi}\right) = \frac{5}{2\pi} - \frac{1}{\pi^2}$ 1

15. $\int (3x+1)\sqrt{4 - 3x - 2x^2}\, dx = -\frac{3}{4}\int(-4x-3)\sqrt{4-3x-2x^2}\, dx - \frac{5}{4}\int\sqrt{4-3x-2x^2}\, dx$ 1

$= -\frac{1}{2}(4-3x-2x^2)^{3/2} - \frac{5}{4}\sqrt{2}\int\sqrt{\left(\frac{\sqrt{41}}{4}\right)^2 - \left(x + \frac{3}{4}\right)^2}\, dx$ 1 + 1

$= -\frac{1}{2}(4-3-2x^2)^{3/2} - \frac{5}{4}\sqrt{2}\left\{\frac{4x+3}{8}\sqrt{\frac{41}{16} - \left(x+\frac{3}{4}\right)^2} + \frac{41}{32}\cdot \sin^{-1}\left(\frac{4x+3}{\sqrt{41}}\right)\right\} + C$

$= -\frac{1}{2}(4-3x-2x^2)^{3/2} - \frac{5}{4}\left\{\frac{4x+3}{8}\sqrt{4-3x-2x^2} + \frac{41\sqrt{2}}{32}\cdot \sin^{-1}\left(\frac{4x+3}{\sqrt{41}}\right)\right\} + C$ 1

16. The differential equation can be re-written as:

$\frac{dy}{dx} = \frac{x-y}{x+y}$, put $y = vx, \frac{dy}{dx} = v + x\frac{dv}{dx}$ $\frac{1}{2} + \frac{1}{2}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{1-v}{1+v} \Rightarrow \frac{1+v}{1-2v-v^2} dv = \frac{1}{x} dx$ 1

integrating we get

$\Rightarrow \frac{1}{2}\int \frac{2V+2}{V^2+2V-1} dv = -\int \frac{1}{x} dx = \frac{1}{2}\log|V^2 + 2V - 1| = -\log x + \log C$ $1\frac{1}{2}$

$\therefore$ Solution of the differential equation is:

$\frac{1}{2}\log\left|\frac{y^2}{x^2} + \frac{2y}{x} - 1\right| = \log C - \log x$ or, $y^2 + 2xy - x^2 = C^2$ $\frac{1}{2}$

17. Let radius of any of the circle touching co-ordinate axes in the second quadrant be "a" then centre is (–a, a)

$\therefore$ Equation of the family of circles is:

$(x + a)^2 + (y - a)^2 = a^2,\ a \in R$ $\quad 1\frac{1}{2}$

$\Rightarrow \quad x^2 + y^2 + 2ax - 2ay + a^2 = 0$

Differentiate w.r.t. "x", $2x + 2yy' + 2a - 2ay' = 0 \Rightarrow a = \dfrac{x + yy'}{y' - 1}$ $\quad 1\frac{1}{2}$

$\therefore$ The differential equation is:

$$\left.\begin{aligned} &\left(x + \frac{x + yy'}{y' - 1}\right)^2 + \left(y - \frac{x + yy'}{y' - 1}\right)^2 = \left(\frac{x + yy'}{y' - 1}\right)^2 \\ \Rightarrow &\left(\frac{xy' + yy'}{y' - 1}\right)^2 + \left(\frac{x + y}{y' - 1}\right)^2 = \left(\frac{x + yy'}{y' - 1}\right)^2 \end{aligned}\right\}$$ 1

18. $\sin^{-1} x + \sin^{-1}(1 - x) = \cos^{-1} x \quad \Rightarrow \quad \sin^{-1}(1 - x) = \frac{\pi}{2} - 2\sin^{-1} x$ $\quad 1$

$\Rightarrow 1 - x = \sin\left(\frac{\pi}{2} - 2\sin^{-1} x\right) \Rightarrow 1 - x = \cos(2\sin^{-1} x) \Rightarrow 1 - x = 1 - 2\sin^2(\sin^{-1} x)$ $\quad 1$

$\Rightarrow 1 - x = 1 - 2x^2$ $\quad 1$

Solving we get, $x = 0$ or $x = \frac{1}{2}$ $\quad 1$

OR

From the equation: $\cos^{-1}\frac{x}{a} = \alpha - \cos^{-1}\frac{y}{b}$

$\frac{x}{a} = \cos\left(\alpha - \cos^{-1}\frac{y}{b}\right) \quad \Rightarrow \quad \frac{x}{a} = \cos\alpha \cdot \cos\left(\cos^{-1}\frac{y}{b}\right) + \sin\alpha \cdot \sin\left(\cos^{-1}\frac{y}{b}\right)$ $\quad 1 + 1$

$\Rightarrow \frac{x}{a} = \frac{y \cdot \cos\alpha}{b} + \sin\alpha\sqrt{1 - \frac{y^2}{b^2}} \quad \Rightarrow \quad \frac{x}{a} - \frac{y}{b}\cos\alpha = \sin\alpha\sqrt{1 - \frac{y^2}{b^2}}$ $\quad 1$

Squaring both sides,

$\Rightarrow \left(\frac{x}{a} - y\frac{\cos\alpha}{b}\right)^2 = \left(\sin\alpha\sqrt{1 - \frac{y^2}{b^2}}\right)^2$ $\quad \frac{1}{2}$

$\Rightarrow \frac{x^2}{a^2} - \frac{2xy}{ab} \cdot \cos\alpha + \frac{y^2}{b^2} = \sin^2\alpha.$ $\quad \frac{1}{2}$

19. let ₹ x be invested in first bond

and ₹ y be invested in second bond

then the system of equations is:

$$\left.\begin{aligned} \frac{10x}{100} + \frac{12y}{100} &= 2800 \\ \frac{12x}{100} + \frac{10y}{100} &= 2700 \end{aligned}\right\} \Rightarrow \left.\begin{aligned} 5x + 6y &= 140000 \\ 6x + 5y &= 135000 \end{aligned}\right\}$$ 1

let $A = \begin{bmatrix} 5 & 6 \\ 6 & 5 \end{bmatrix}; X = \begin{bmatrix} x \\ y \end{bmatrix}; B = \begin{bmatrix} 140000 \\ 135000 \end{bmatrix}$

$\therefore\ A \cdot X = B$

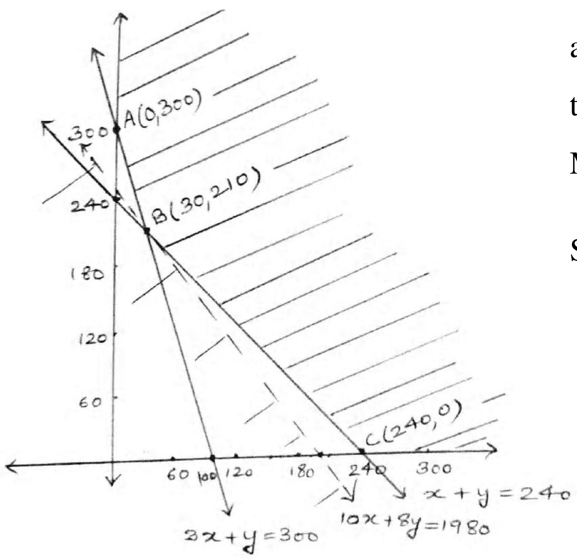
$|A| = -\ 11;\ A^{-1} = \dfrac{1}{-11}\begin{bmatrix} 5 & -6 \\ -6 & 5 \end{bmatrix}$ 1

$$\left.\begin{array}{l} \therefore \text{ Solution is } X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \dfrac{1}{-11}\begin{bmatrix} 5 & -6 \\ -6 & 5 \end{bmatrix}\begin{bmatrix} 140000 \\ 135000 \end{bmatrix} = \begin{bmatrix} 10000 \\ 15000 \end{bmatrix} \\ \therefore x = 10000,\ y = 15000, \therefore \text{ Amount invested} = ₹\ 25000 \end{array}\right\}$$ $\frac{1}{2} + \frac{1}{2}$

Value: caring elders 1

SECTION C

20.



Let x kg of fertilizer A be used

and y kg of fertilizer B be used

then the linear programming problem is:

Minimise cost: z = 10x + 8y 1

$$\left.\begin{array}{l} \text{Subject to } \dfrac{12x}{100} + \dfrac{4y}{100} \geq 12 \Rightarrow 3x + y \geq 300 \\ \qquad\qquad \dfrac{5x}{100} + \dfrac{5y}{100} \geq 12 \Rightarrow x + y \geq 240 \\ \qquad\qquad x, y \geq 0 \end{array}\right\}$$ 2

Correct Graph $1\frac{1}{2}$

Value of Z at corners of the unbounded region ABC:

Corner	Value of Z
A (0, 300)	₹ 2400
B(30, 210)	₹ 1980 (Minimum)
C(240, 0)	₹ 2400

1

The region of 10x + 8y < 1980 or 5x + 4y < 990 has no point in common to the feasible region. Hence, minimum cost = ₹ 1980 at x = 30 and y = 210 $\frac{1}{2}$

21. Let X = Number of bad oranges out of 4 drawn = 0, 1, 2, 3, 4 1

P = Probability of a bad orange = $\frac{1}{5}, q = 1 - p = \frac{4}{5}$ $\frac{1}{2}$

$\therefore$ Probability distribution is:

X:	0	1	2	3	4
P(X):	${}^4C_0\left(\frac{4}{5}\right)^4 = \frac{256}{625}$	${}^4C_1\frac{1}{5}\left(\frac{4}{5}\right)^3$	${}^4C_2\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^2$	${}^4C_3\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)$	${}^4C_4\left(\frac{1}{5}\right)^4$
		$= \frac{256}{625}$	$= \frac{96}{625}$	$= \frac{16}{625}$	$= \frac{1}{625}$

$2\frac{1}{2}$

Mean (μ) = $\Sigma X.P(X) = 0\times\frac{256}{625}+1\times\frac{256}{625}+2\times\frac{96}{625}+3\times\frac{16}{625}+4\times\frac{1}{625}=\frac{4}{5}$ 1

Variance (σ^2) $= \Sigma x^2.P(x) - [\Sigma x.P(x)]^2$

$$= 0\times\frac{256}{625}+\frac{1\times 256}{625}+\frac{4\times 96}{625}+\frac{9\times 16}{625}+\frac{16}{625}-\left(\frac{4}{5}\right)^2=\frac{16}{25}$$ 1

22. Line through 'P' and perpendicular to plane is:

$\vec{r}=(2\hat{i}+3\hat{j}+4\hat{k})+\lambda(2\hat{i}+\hat{j}+3\hat{k})$ 1

General point on line is: $\vec{r}=(2+2\lambda)\hat{i}+(3+\lambda)\hat{j}+(4+3\lambda)\hat{k}$

For some $\lambda \in R$, $\vec{r}$ is the foot of perpendicular, say Q, from P to the plane, since it lies on plane

$\therefore \quad [(2+2\lambda)\hat{i}+(3+\lambda)\hat{j}+(4+3\lambda)\hat{k}]\cdot(2\hat{i}+\hat{j}+3\hat{k})-26=0$

$\Rightarrow \quad 4+4\lambda+3+\lambda+12+9\lambda-26=0 \;\Rightarrow\; \lambda=\frac{1}{2}$ $1\frac{1}{2}$

$\therefore$ Foot of perpendicular is $Q\left(3\hat{i}+\frac{7}{2}\hat{j}+\frac{11}{2}\hat{k}\right)$ 1

let $P'(a\hat{i}+b\hat{j}+c\hat{k})$ be the image of P in the plane then Q is mid point of PP′

$\therefore \quad Q\left(\frac{a+2}{2}\hat{i}+\frac{b+3}{2}\hat{j}+\frac{c+4}{2}\hat{k}\right)=Q\left(3\hat{i}+\frac{7}{2}\hat{j}+\frac{11}{2}\hat{k}\right)$ 1

$\Rightarrow \quad \frac{a+2}{2}=3, \frac{b+3}{2}=\frac{7}{2}, \frac{c+4}{2}=\frac{11}{2} \;\Rightarrow\; a=4, b=4, c=7 \;\therefore P'(4\hat{i}+4\hat{j}+7\hat{k})$ 1

Perpendicular distance of P from plane = $PQ=\sqrt{(2-3)^2+\left(3-\frac{7}{2}\right)^2+\left(4-\frac{11}{2}\right)^2}=\sqrt{\frac{7}{2}}$ $\frac{1}{2}$

23. Commutative: For any elements a, b $\in$ A

a * b = a + b + ab = b + a + ba = b * a. Hence * is commutative $1\frac{1}{2}$

Associative: For any three elements a, b, c, $\in$ A

a * (b * c) = a * (b + c + bc) = a + b + c + bc + ab + ac + abc
(a * b) * c = (a + b + ab) * c = a + b + ab + c + ac + bc + abc $1\frac{1}{2}$

$\therefore$ a * (b * c) = (a * b) * c, Hence * is Associative.

Identity element: let e $\in$ A be the identity element them a * e = e * a = a

$\Rightarrow$ a + e + ae = e + a + ea = a $\Rightarrow$ e (1 + a) = 0, as a $\neq$ – 1

e = 0 is the identity element $1\frac{1}{2}$

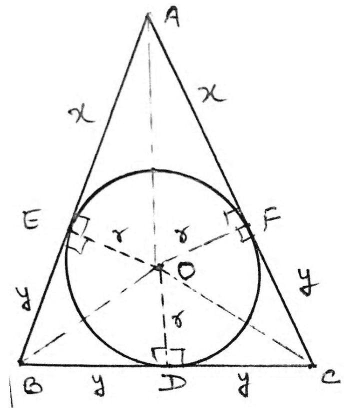
Invertible: let a, b $\in$ A so that 'b' is inverse of a

$\therefore$ a * b = b * a = e

$\Rightarrow$ a + b + ab = b + a + ba = 0

As $a \neq -1$, $b=\frac{-a}{1+a} \in A$. Hence every element of A is invertible $1\frac{1}{2}$

24. Correct Figure 1



Let ΔABC be isosceles with inscribed circle of radius 'r' touching sides AB, AC and BC at E, F and D respectively.

let AE = AF = x, BE = BD = y, CF = CD = y then

area (ΔABC) = ar(ΔAOB) + ar(ΔAOC) + ar(ΔBOC)

$$\Rightarrow \frac{1}{2}.2y\left(r+\sqrt{r^2+x^2}\right) = \frac{1}{2}\{2yr+2(x+y)r\} \Rightarrow x = \frac{2r^2y}{y^2-r^2}$$ 1

Then,

$$P(\text{Perimeter of } \Delta ABC) = 2x + 4y = \frac{4r^2y}{y^2-r^2} + 4y$$ 1

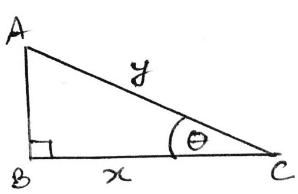
$$\frac{dP}{dy} = \frac{-4r^2(r^2+y^2)}{(y^2-r^2)^2} + 4 \text{ and } \frac{dP}{dy} = 0 \Rightarrow y = \sqrt{3}r$$ $1 + \frac{1}{2}$

$$\left.\frac{d^2P}{dy^2}\right]_{y=\sqrt{3}r} = \frac{4r^2y(2y^2+6r^2)}{(y^2-r^2)^3} = \frac{6\sqrt{3}}{r} > 0$$ $\frac{1}{2}$

$\therefore$ Perimeter is least iff $y = \sqrt{3}r$ and least perimeter is

$$P = 4y + \frac{4r^2y}{y^2-r^2} = 4\sqrt{3}r + \frac{4r^2\sqrt{3}r}{2r^2} = 6\sqrt{3}r$$ 1

OR



let ABC be the right triangle with $\angle B = 90°$

$\angle ACB = \theta$, AC = y, BC = x, x + y = k (constant)

$$A \text{ (Area of triange)} = \frac{1}{2}.BC.AB = \frac{1}{2}.x\sqrt{y^2-x^2}$$ $1\frac{1}{2}$

$$\text{let } z = A^2 = \frac{1}{4}x^2(y^2-x^2) = \frac{1}{4}x^2\{(k-x)^2-x^2\} = \frac{1}{4}(x^2k^2-2kx^3)$$ 1

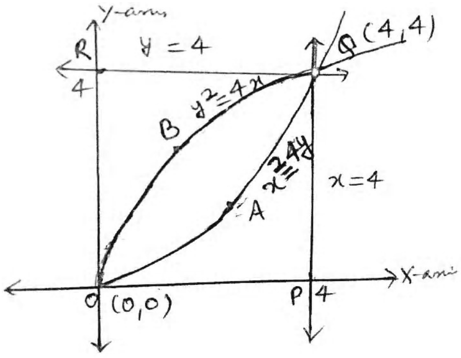
$$\frac{dz}{dx} = \frac{1}{4}(2xk^2-6kx^2) \text{ and } \frac{dz}{dx} = 0 \Rightarrow x = \frac{k}{3},\ y = k - x = \frac{2k}{3}$$ 1+1

$$\left.\frac{d^2z}{dx^2}\right]_{x=\frac{k}{3}} = \left.\frac{1}{4}(2k^2-12kx)\right]_{x=\frac{k}{3}} = -\frac{k^2}{2} < 0$$ $\frac{1}{2}$

$\therefore$ z and area of ΔABC is max at $x = \frac{k}{3}$

$$\text{and, } \cos\theta = \frac{x}{y} = \frac{k}{3}.\frac{3}{2k} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$ 1

25. Point of intersection of $y^2 = 4x$ and $x^2 = 4y$ are (0, 0) and (4, 4);



Correct Graph $1\frac{1}{2}$

$$\text{are (OAQBO)} = \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right) dx$$ 1

$$= \left\{\frac{4}{3}x^{3/2} - \frac{x^3}{12}\right\}\Big]_0^4$$

$$= \frac{32}{3} - \frac{16}{3} = \frac{16}{3}$$ $\frac{1}{2}$

$$\text{area (OPQAO)} = \int_0^4 \frac{x^2}{4} dx = \frac{1}{12}x^3\Big]_0^4 = \frac{16}{3}$$ $1\frac{1}{2}$

$$\text{area (OBQRO)} = \int_0^4 \frac{y^2}{4} dy = \frac{1}{12}y^3\Big]_0^4 = \frac{16}{3}$$ $1\frac{1}{2}$

Hence the areas of the three regions are equal.

26. $$\begin{vmatrix} 1 & 1 & 1 \\ 1+\cos A & 1+\cos B & 1+\cos C \\ \cos^2 A + \cos A & \cos^2 B + \cos B & \cos^2 C + \cos C \end{vmatrix} = 0$$

Apply $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$

$$\Leftrightarrow \begin{vmatrix} 1 & 0 & 0 \\ 1+\cos A & \cos B - \cos A & \cos C - \cos A \\ \cos^2 A + \cos A & (\cos B - \cos A)(\cos B + \cos A + 1) & (\cos C - \cos A)(\cos C + \cos A + 1) \end{vmatrix} = 0$$ 3

Taking (cos B – cos A), (cos C – cos A) common from C_2 & C_3

$$\Leftrightarrow (\cos B - \cos A)(\cos C - \cos A)\begin{vmatrix} 1 & 0 & 0 \\ 1+\cos A & 1 & 1 \\ \cos^2 A + \cos A & \cos B + \cos A + 1 & \cos C + \cos A + 1 \end{vmatrix} = 0$$ 1

Expand along R_1

$\Leftrightarrow$ (cos B – cos A) (cos C – cos A) (cos C – cos B) = 0 1

$\Leftrightarrow$ cos A = cos B $\Leftrightarrow$ A = B $\Leftrightarrow$ ΔABC is an isosceles triangle 1

or or

cos B = cos C B = C

or or

cos C = cos A C = A

OR

let the cost of one pen of veriety 'A', 'B' and 'C' be ₹ x. ₹ y and ₹ z respectively then the system of equations is:

$$\left.\begin{aligned} x + y + z &= 21 \\ 4x + 3y + 2z &= 60 \\ 6x + 2y + 3z &= 70 \end{aligned}\right\} \qquad 1\frac{1}{2}$$

Matrix form of the system is:

$$A \cdot X = B, \text{ where } A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; B = \begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix} \qquad \frac{1}{2}$$

$|A| = (5) - 1\ (0) + 1\ (-10) = -5$ 1

co-factors of the matrix A are:

$$\left.\begin{matrix} C_{11} = 5; & C_{21} = -1; & C_{31} = -1 \\ C_{12} = 0; & C_{22} = -3 & C_{32} = 2 \\ C_{13} = -10; & C_{23} = 4; & C_{33} = -1 \end{matrix}\right\} \qquad 2$$

$$\therefore\ A^{-1} = \frac{1}{|A|} \text{Adj } A = \frac{1}{-5}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix} \qquad \frac{1}{2}$$

Solution of the matrix equation is $X = A^{-1} B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}\begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix} \therefore x = 5, y = 8, z = 8 \qquad \frac{1}{2}$$

QUESTION PAPER CODE 65/2/C

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. 2 — 1

2. No. of possible matrices = 3^4 or 81 — 1

3. $(x + 3)2x - (-2)(-3x) = 8$ — $\frac{1}{2}$

$x = 2$ — $\frac{1}{2}$

4. $\dfrac{2(2\vec{a}+3\vec{b})+1(3\vec{a}-2\vec{b})}{2+1}$ — $\frac{1}{2}$

$= \dfrac{7}{3}\vec{a}+\dfrac{4}{3}\vec{b}$ (or enternal division may also be considered) — $\frac{1}{2}$

5. $\dfrac{x}{3}+\dfrac{y}{-4}+\dfrac{z}{2} = 1$ — $\frac{1}{2}$

$\Rightarrow \vec{r}\cdot(4\hat{i}-3\hat{j}+6\hat{k}) = 12$ or $\vec{r}\cdot\left(\dfrac{\hat{i}}{3}-\dfrac{\hat{j}}{4}+\dfrac{\hat{k}}{2}\right)=1$ — $\frac{1}{2}$

6. $\begin{pmatrix} 2 & 5 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -1 & -1 \end{pmatrix}$ — $\frac{1}{2}+\frac{1}{2}$

SECTION B

7. $y^2 = ax^3 + b \Rightarrow 2y\dfrac{dy}{dx} = 3ax^2 \therefore \dfrac{dy}{dx} = \dfrac{3a}{2}\dfrac{x^2}{y}$ — 1

Slope of tangent at (2, 3) = $\left.\dfrac{dy}{dx}\right]_{(2,\,3)} = \dfrac{3a}{2}\cdot\dfrac{4}{3} = 2a$ — 1

Comparing with slope of tangent $y = 4x - 5$, we get, $2a = 4 \therefore \boxed{a = 2}$ — 1

Also (2, 3) lies on the curve $\therefore 9 = 8a + b$, put $a = 2$, we get $b = -7$ — 1

8. Equation of line through A(3, 4, 1) and B(5, 1, 6)

$\dfrac{x-3}{2} = \dfrac{y-4}{-3} = \dfrac{z-1}{5} = k\text{(say)}$ — 1

General point on the line:

$x = 2k + 3,\ y = -3k + 4,\ z = 5k + 1$ — $\frac{1}{2}$

line crosses xz plane i.e. $y = 0$ if $-3k + 4 = 0$

$\therefore \quad k = \dfrac{4}{3}$ — 1

Co-ordinate of required point $\left(\frac{17}{3}, 0, \frac{23}{3}\right)$ $\frac{1}{2}$

Angle, which line makes with xz plane:

$$\sin\theta = \left|\frac{2\,(0) + (-3)\,(1) + 5(0)}{\sqrt{4+9+25}\,\sqrt{1}}\right| = \frac{3}{\sqrt{38}} \Rightarrow \theta = \sin^{-1}\left(\frac{3}{\sqrt{38}}\right) \qquad 1$$

9. $$\int (3x+1)\sqrt{4-3x-2x^2}\,dx = -\frac{3}{4}\int(-4x-3)\sqrt{4-3x-2x^2}\,dx - \frac{5}{4}\int\sqrt{4-3x-2x^2}\,dx \qquad 1$$

$$= -\frac{1}{2}(4-3x-2x^2)^{3/2} - \frac{5}{4}\sqrt{2}\int\sqrt{\left(\frac{\sqrt{41}}{4}\right)^2 - \left(x+\frac{3}{4}\right)^2}\,dx \qquad 1+1$$

$$= -\frac{1}{2}(4-3-2x^2)^{3/2} - \frac{5}{4}\sqrt{2}\left\{\frac{4x+3}{8}\sqrt{\frac{41}{16} - \left(x+\frac{3}{4}\right)^2} + \frac{41}{32}\cdot\sin^{-1}\left(\frac{4x+3}{\sqrt{41}}\right)\right\} + C$$

$$= -\frac{1}{2}(4-3x-2x^2)^{3/2} - \frac{5}{4}\left\{\frac{4x+3}{8}\sqrt{4-3x-2x^2} + \frac{41\sqrt{2}}{32}\cdot\sin^{-1}\left(\frac{4x+3}{\sqrt{41}}\right)\right\} + C \qquad 1$$

10. Let $\vec{d}_1$ & $\vec{d}_2$ be the two diagonal vectors:

$\therefore \quad \vec{d}_1 = 4\hat{i} - 2\hat{j} - 2\hat{k},\ \vec{d}_2 = -6\hat{j} - 8\hat{k}$ $\frac{1}{2} + \frac{1}{2}$

or $\quad \vec{d}_2 = 6\hat{j} + 8\hat{k}$

Unit vectors parallel to the diagonals are:

$\hat{d}_1 = \frac{2}{\sqrt{6}}\hat{i} - \frac{1}{\sqrt{6}}\hat{j} - \frac{1}{\sqrt{6}}\hat{k}$ $\frac{1}{2}$

$\hat{d}_2 = -\frac{3}{5}\hat{j} - \frac{4}{5}\hat{k} \quad \left(\text{or } \hat{d}_2 = \frac{3}{5}\hat{j} + \frac{4}{5}\hat{k}\right)$ $\frac{1}{2}$

$$\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -2 & -2 \\ 0 & -6 & -8 \end{vmatrix} = 4\hat{i} + 32\hat{j} - 24\hat{k} \qquad 1$$

Area of parallelogram $= \frac{1}{2}\left|\vec{d}_1 \times \vec{d}_2\right| = \sqrt{404}$ or $2\sqrt{101}$ sq. units 1

11. Let radius of any of the circle touching co-ordinate axes in the second quadrant be "a" then centre is (–a, a)

$\therefore$ Equation of the family of circles is:

$(x + a)^2 + (y - a)^2 = a^2,\ a \in R$ $1\frac{1}{2}$

$\Rightarrow \quad x^2 + y^2 + 2ax - 2ay + a^2 = 0$

Differentiate w.r.t. "x", $2x + 2yy' + 2a - 2ay' = 0 \Rightarrow a = \frac{x + yy'}{y' - 1}$ $1\frac{1}{2}$

$\therefore$ The differential equation is:

$$\left.\begin{array}{l}\left(x+\dfrac{x+yy'}{y'-1}\right)^2+\left(y-\dfrac{x+yy'}{y'-1}\right)^2=\left(\dfrac{x+yy'}{y'-1}\right)^2 \\ \Rightarrow\left(\dfrac{xy'+yy'}{y'-1}\right)^2+\left(\dfrac{x+y}{y'-1}\right)^2=\left(\dfrac{x+yy'}{y'-1}\right)^2\end{array}\right\}$$ 1

12. Let X = Amount he wins then x = ₹ 5, 4, 3, – 3 1

P = Probability of getting a no. >4 = $\frac{1}{3}$, q = $1-p=\frac{2}{3}$ $\frac{1}{2}$

X:	5	4	3	–3
P(x)	$\frac{1}{3}$	$\frac{2}{3}\cdot\frac{1}{3}=\frac{2}{9}$	$\left(\frac{2}{3}\right)^2\cdot\frac{1}{3}=\frac{4}{27}$	$\left(\frac{2}{3}\right)^3=\frac{8}{27}$

2

Expected amount he wins $=\Sigma\,XP(X)=\frac{5}{3}+\frac{8}{9}+\frac{12}{27}-\frac{24}{27}$

$= ₹\,\frac{19}{9}$ or $₹\,2\frac{1}{9}$ $\frac{1}{2}$

OR

E_1 = Event that all balls are white,
E_2 = Event that 3 balls are white and 1 ball is non white
E_3 = Event that 2 balls are white and 2 balls are non-white
A = Event that 2 balls drawn without replacement are white 1

$P(E_1)=P(E_2)=P(E_3)=\frac{1}{3}$ $\frac{1}{2}$

$P(A/E_1)=1$, $P(A/E_2)=\frac{3}{4}\cdot\frac{2}{3}=\frac{1}{2}$, $P(A/E_3)=\frac{2}{4}\cdot\frac{1}{3}=\frac{1}{6}$ $1\frac{1}{2}$

$$P(E_1/A)=\frac{1.\frac{1}{3}}{1\cdot\frac{1}{3}+\frac{1}{3}\cdot\frac{1}{2}+\frac{1}{3}\cdot\frac{1}{6}}=\frac{3}{5}$$ 1

13. Let ₹ x be invested in first bond

and ₹ y be invested in second bond

then the system of equations is:

$$\left.\begin{array}{l}\dfrac{10x}{100}+\dfrac{12y}{100}=2800 \\ \dfrac{12x}{100}+\dfrac{10y}{100}=2700\end{array}\right\}\Rightarrow\left.\begin{array}{l}5x+6y=140000 \\ 6x+5y=135000\end{array}\right\}$$ 1

let $A=\begin{bmatrix}5&6\\6&5\end{bmatrix}$; $X=\begin{bmatrix}x\\y\end{bmatrix}$; $B=\begin{bmatrix}140000\\135000\end{bmatrix}$

$\therefore$ A·X = B

$|A|=-11$; $A^{-1}=\frac{1}{-11}\begin{bmatrix}5&-6\\-6&5\end{bmatrix}$ 1

$\therefore$ Solution is $X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{-11}\begin{bmatrix} 5 & -6 \\ -6 & 5 \end{bmatrix}\begin{bmatrix} 140000 \\ 135000 \end{bmatrix} = \begin{bmatrix} 10000 \\ 15000 \end{bmatrix}$ $\quad \frac{1}{2}+\frac{1}{2}$

$\therefore x = 10000,\ y = 15000, \therefore$ Amount invested = ₹ 25000

Value: caring elders 1

14. let y = u + v, $u = x^{\sin x}$, $v = (\sin x)^{\cos x}$

$\log u = \sin x.\log x \Rightarrow \frac{du}{dx} = x^{\sin x}\cdot\left\{\cos x.\log x + \frac{\sin x}{x}\right\}$ $\quad \frac{1}{2}+1$

$\log v = \cos x.\log(\sin x) \Rightarrow \frac{dv}{dx} = (\sin x)^{\cos x}\cdot\{\cos x\cdot\cot x - \sin x\cdot\log(\sin x)\}$ $\quad \frac{1}{2}+1$

$\frac{dy}{dx} = \frac{du}{dx}+\frac{dv}{dx} = x^{\sin x}\cdot\left\{\cos x\cdot\log x + \frac{\sin x}{x}\right\} + (\sin x)^{\cos x}\{\cos x\cdot\cot x - \sin x\cdot\log(\sin x)\}$ $\quad \frac{1}{2}+\frac{1}{2}$

OR

$\frac{dy}{dx} = \frac{-2\sin(\log x)}{x} + \frac{3\cos(\log x)}{x}$ 1

$\Rightarrow \quad x\frac{dy}{dx} = -2\sin(\log x) + 3\cos(\log x)$, differentiate w.r.t 'x' $\quad \frac{1}{2}$

$\Rightarrow \quad x\frac{d^2y}{dx^2} + \frac{dy}{dx} = \frac{-2\cos(\log x)}{x} - \frac{3\sin(\log x)}{x}$ 2

$\Rightarrow \quad x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -y \Rightarrow x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ $\quad \frac{1}{2}$

15. $\sin^{-1}x + \sin^{-1}(1-x) = \cos^{-1}x \Rightarrow \sin^{-1}(1-x) = \frac{\pi}{2} - 2\sin^{-1}x$ 1

$\Rightarrow 1 - x = \sin\left(\frac{\pi}{2} - 2\sin^{-1}x\right) \Rightarrow 1 - x = \cos(2\sin^{-1}x) \Rightarrow 1 - x = 1 - 2\sin^2(\sin^{-1}x)$ 1

$\Rightarrow 1 - x = 1 - 2x^2$ 1

Solving we get, $x = 0$ or $x = \frac{1}{2}$ 1

OR

From the equation: $\cos^{-1}\frac{x}{a} = \alpha - \cos^{-1}\frac{y}{b}$

$\frac{x}{a} = \cos\left(\alpha - \cos^{-1}\frac{y}{b}\right) \Rightarrow \frac{x}{a} = \cos\alpha\cdot\cos\left(\cos^{-1}\frac{y}{b}\right) + \sin\alpha\cdot\sin\left(\cos^{-1}\frac{y}{b}\right)$ 1 + 1

$\Rightarrow \frac{x}{a} = \frac{y\cdot\cos\alpha}{b} + \sin\alpha\sqrt{1-\frac{y^2}{b^2}} \Rightarrow \frac{x}{a} - \frac{y}{b}\cos\alpha = \sin\alpha\sqrt{1-\frac{y^2}{b^2}}$ 1

Squaring both sides,

$\Rightarrow \left(\frac{x}{a} - y\frac{\cos\alpha}{b}\right)^2 = \left(\sin\alpha\sqrt{1-\frac{y^2}{b^2}}\right)^2$ $\quad \frac{1}{2}$

$\Rightarrow \frac{x^2}{a^2} - \frac{2xy}{ab}\cdot\cos\alpha + \frac{y^2}{b^2} = \sin^2\alpha.$ $\quad \frac{1}{2}$

16. $\frac{dx}{dt} = 2a\cos 2t\,(1+\cos 2t) - 2a\sin 2t\cdot\sin 2t$ $\quad 1\frac{1}{2}$

$\frac{dy}{dt} = -2b\sin 2t\,(1-\cos 2t) + 2b\cos 2t\cdot\sin 2t$ $\quad 1$

$\left.\frac{dy}{dx}\right]_{t=\frac{\pi}{4}} = \left.\frac{2b\cos 2t\cdot\sin 2t - 2b\sin 2t\,(1-\cos 2t)}{2a\cos 2t\,(1+\cos 2t) - 2a\sin 2t\cdot\sin 2t}\right]_{t=\frac{\pi}{4}} = \frac{b}{a}$ $\quad \frac{1}{2} + 1$

17. The differential equation can be re-written as:

$\frac{dy}{dx} = \frac{x-y}{x+y}$, put $y = vx$, $\frac{dy}{dx} = v + x\frac{dv}{dx}$ $\quad \frac{1}{2} + \frac{1}{2}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{1-v}{1+v} \Rightarrow \frac{1+v}{1-2v-v^2}\,dv = \frac{1}{x}\,dx$ $\quad 1$

integrating we get

$\Rightarrow \frac{1}{2}\int \frac{2V+2}{V^2+2V-1}\,dv = -\int \frac{1}{x}\,dx = \frac{1}{2}\log|V^2+2V-1| = -\log x + \log C$ $\quad 1\frac{1}{2}$

$\therefore$ Solution of the differential equation is:

$\frac{1}{2}\log\left|\frac{y^2}{x^2} + \frac{2y}{x} - 1\right| = \log C - \log x$ or, $y^2 + 2xy - x^2 = C^2$ $\quad \frac{1}{2}$

18. Let $I = \int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x}\,dx$, Also $I = \int_0^{\pi/2} \frac{\sin^2\left(\frac{\pi}{2} - x\right)}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)}\,dx = \int_0^{\pi/2} \frac{\cos^2 x}{\cos x + \sin x}\,dx$ $\quad 1$

Adding to get, $2I = \int_0^{\pi/2} \frac{1}{\sin x + \cos x}\,dx = \frac{1}{\sqrt{2}}\int_0^{\pi/2} \frac{1}{\cos(x - \pi/4)}\,dx$ $\quad \frac{1}{2} + 1$

$\Rightarrow 2I = \frac{1}{\sqrt{2}}\int_0^{\pi/2} \sec(x - \pi/4)\,dx = \frac{1}{\sqrt{2}}\log\left|\sec\left(x - \frac{\pi}{4}\right) + \tan\left(x - \frac{\pi}{4}\right)\right|_0^{\pi/2}$ $\quad 1$

$\Rightarrow 2I = \frac{1}{\sqrt{2}}\left\{\log\left|\sqrt{2}+1\right| - \log\left|\sqrt{2}-1\right|\right\}$

$\Rightarrow I = \frac{1}{2\sqrt{2}}\left\{\log\left|\sqrt{2}+1\right| - \log\left|\sqrt{2}-1\right|\right.$ or $\frac{1}{2\sqrt{2}}\log\left|\frac{\sqrt{2}+1}{\sqrt{2}-1}\right|$ $\quad \frac{1}{2}$

OR

$\int_0^{3/2} |x\cos \pi x|\,dx = \int_0^{1/2} x\cos \pi x\,dx - \int_{1/2}^{3/2} x\cos \pi x\,dx$ $\quad 1\frac{1}{2}$

$= \left\{\frac{x\sin \pi x}{\pi} + \frac{\cos \pi x}{\pi^2}\right\}_0^{1/2} - \left\{\frac{x\sin \pi x}{\pi} + \frac{\cos \pi x}{\pi^2}\right\}_{1/2}^{3/2}$ $\quad 1\frac{1}{2}$

$= \frac{1}{2\pi} - \frac{1}{\pi^2} - \left(-\frac{3}{2\pi} - \frac{1}{2\pi}\right) = \frac{5}{2\pi} - \frac{1}{\pi^2}$ $\quad 1$

19. Let $x^2 = t \therefore \dfrac{x^2}{x^4 + x^2 - 2} = \dfrac{x^2}{(x^2-1)(x^2+2)} = \dfrac{t}{(t-1)(t+2)} = \dfrac{A}{t-1} + \dfrac{B}{t+2}$ 1

Solving for A and B to get, $A = \dfrac{1}{3}, B = \dfrac{2}{3}$ 1

$\displaystyle\int \frac{x^2}{x^4 + x^2 - 2}\,dx = \frac{1}{3}\int \frac{1}{x^2-1}\,dx + \frac{2}{3}\int \frac{1}{x^2+2}\,dx = \frac{1}{6}\log\left|\frac{x-1}{x+1}\right| + \frac{\sqrt{2}}{3}\tan^{-1}\frac{x}{\sqrt{2}} + C$ 1 + 1

SECTION C

20. $$\begin{vmatrix} 1 & 1 & 1 \\ 1+\cos A & 1+\cos B & 1+\cos C \\ \cos^2 A + \cos A & \cos^2 B + \cos B & \cos^2 C + \cos C \end{vmatrix} = 0$$

Apply $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$

$$\Leftrightarrow \begin{vmatrix} 1 & 0 & 0 \\ 1+\cos A & \cos B - \cos A & \cos C - \cos A \\ \cos^2 A + \cos A & (\cos B - \cos A)(\cos B + \cos A + 1) & (\cos C - \cos A)(\cos C + \cos A + 1) \end{vmatrix} = 0$$ 3

Taking (cos B – cos A), (cos C – cos A) common from C_2 & C_3

$$\Leftrightarrow (\cos B - \cos A)(\cos C - \cos A)\begin{vmatrix} 1 & 0 & 0 \\ 1+\cos A & 1 & 1 \\ \cos^2 A + \cos A & \cos B + \cos A + 1 & \cos C + \cos A + 1 \end{vmatrix} = 0$$ 1

Expand along R_1

$\Leftrightarrow (\cos B - \cos A)(\cos C - \cos A)(\cos C - \cos B) = 0$ 1

$\Leftrightarrow \cos A = \cos B$ $\quad\Leftrightarrow A = B$ $\quad\Leftrightarrow \Delta ABC$ is an isosceles triangle 1

or $\quad$ or

$\cos B = \cos C$ $\quad B = C$

or $\quad$ or

$\cos C = \cos A$ $\quad C = A$

OR

let the cost of one pen of veriety ‘A’, ‘B’ and ‘C’ be ₹ x. ₹ y and ₹ z respectively then the system of equations is:

$$\left.\begin{aligned} x + y + z &= 21 \\ 4x + 3y + 2z &= 60 \\ 6x + 2y + 3z &= 70 \end{aligned}\right\}$$ $1\frac{1}{2}$

Matrix form of the system is:

$A \cdot X = B$, where $A = \begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix}; X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}; B = \begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix}$ $\frac{1}{2}$

$|A| = (5) - 1\,(0) + 1\,(-10) = -5$ 1

co-factors of the matrix A are:

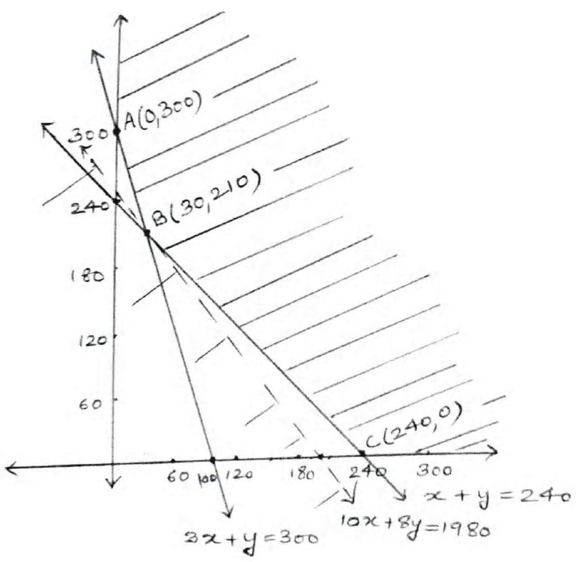
$$\left.\begin{matrix} C_{11}=5; & C_{21}=-1; & C_{31}=-1 \\ C_{12}=0; & C_{22}=-3 & C_{32}=2 \\ C_{13}=-10; & C_{23}=4; & C_{33}=-1 \end{matrix}\right\}$$ 2

$$\therefore\ A^{-1} = \frac{1}{|A|}\text{Adj } A = \frac{1}{-5}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}$$ $\frac{1}{2}$

Solution of the matrix equation is $X = A^{-1} B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}\begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix} \therefore x = 5,\ y = 8,\ z = 8$$ $\frac{1}{2}$

21.



Let x kg of fertilizer A be used

and y kg of fertilizer B be used

then the linear programming problem is:

Minimise cost: $z = 10x + 8y$ 1

Subject to $$\left.\begin{matrix} \frac{12x}{100} + \frac{4y}{100} \geq 12 \Rightarrow 3x + y \geq 300 \\ \frac{5x}{100} + \frac{5y}{100} \geq 12 \Rightarrow x + y \geq 240 \\ x, y \geq 0 \end{matrix}\right\}$$ 2

Correct Graph $1\frac{1}{2}$

Value of Z at corners of the unbounded region ABC:

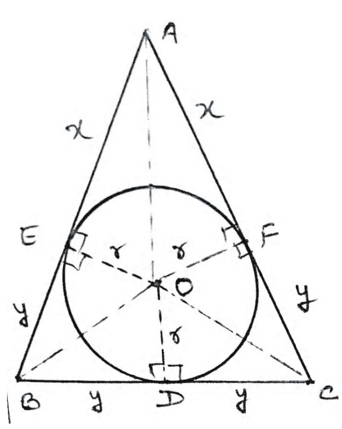
Corner	Value of Z
A (0, 300)	₹ 2400
B(30, 210)	₹ 1980 (Minimum)
C(240, 0)	₹ 2400

1

The region of $10x + 8y < 1980$ or $5x + 4y < 990$ has no point in common to the feasible region. Hence, minimum cost = ₹ 1980 at x = 30 and y = 210 $\frac{1}{2}$

22.

Correct Figure 1



Let ΔABC be isosceles with inscribed circle of radius 'r' touching sides AB, AC and BC at E, F and D respectively.

let AE = AF = x, BE = BD = y, CF = CD = y then

area $(\Delta ABC) = ar(\Delta AOB) + ar(\Delta AOC) + ar(\Delta BOC)$

$$\Rightarrow \frac{1}{2}.2y\left(r + \sqrt{r^2 + x^2}\right) = \frac{1}{2}\{2yr + 2(x+y)r\} \Rightarrow x = \frac{2r^2y}{y^2 - r^2}$$ 1

Then,

$$P(\text{Perimeter of } \Delta ABC) = 2x + 4y = \frac{4r^2y}{y^2 - r^2} + 4y \qquad 1$$

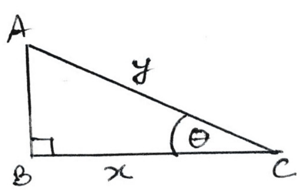
$$\frac{dP}{dy} = \frac{-4r^2(r^2+y^2)}{(y^2-r^2)^2} + 4 \text{ and } \frac{dP}{dy} = 0 \Rightarrow y = \sqrt{3}r \qquad 1 + \frac{1}{2}$$

$$\left.\frac{d^2P}{dy^2}\right]_{y=\sqrt{3}r} = \frac{4r^2y(2y^2+6r^2)}{(y^2-r^2)^3} = \frac{6\sqrt{3}}{r} > 0 \qquad \frac{1}{2}$$

$\therefore$ Perimeter is least iff $y = \sqrt{3}r$ and least perimeter is

$$P = 4y + \frac{4r^2y}{y^2-r^2} = 4\sqrt{3}r + \frac{4r^2\sqrt{3}r}{2r^2} = 6\sqrt{3}r \qquad 1$$

OR



let ABC be the right triangle with $\angle B = 90°$

$\angle ACB = \theta$, AC = y, BC = x, x + y = k (constant)

$$A \text{ (Area of triange)} = \frac{1}{2}. BC. AB = \frac{1}{2}. x\sqrt{y^2 - x^2} \qquad 1\frac{1}{2}$$

$$\text{let } z = A^2 = \frac{1}{4}x^2(y^2-x^2) = \frac{1}{4}x^2\{(k-x)^2 - x^2\} = \frac{1}{4}(x^2k^2 - 2kx^3) \qquad 1$$

$$\frac{dz}{dx} = \frac{1}{4}(2xk^2 - 6kx^2) \text{ and } \frac{dz}{dx} = 0 \Rightarrow x = \frac{k}{3}, y = k - x = \frac{2k}{3} \qquad 1+1$$

$$\left.\frac{d^2z}{dx^2}\right]_{x=\frac{k}{3}} = \left.\frac{1}{4}(2k^2 - 12kx)\right]_{x=\frac{k}{3}} = -\frac{k^2}{2} < 0 \qquad \frac{1}{2}$$

$\therefore$ z and area of ΔABC is max at $x = \frac{k}{3}$

$$\text{and, } \cos\theta = \frac{x}{y} = \frac{k}{3}.\frac{3}{2k} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3} \qquad 1$$

23. Let X = Number of bad oranges out of 4 drawn = 0, 1, 2, 3, 4 1

P = Probability of a bad orange = $\frac{1}{5}, q = 1 - p = \frac{4}{5}$ $\frac{1}{2}$

$\therefore$ Probability distribution is:

X:	0	1	2	3	4
P(X):	${}^4C_0\left(\frac{4}{5}\right)^4 = \frac{256}{625}$	${}^4C_1\frac{1}{5}\left(\frac{4}{5}\right)^3$	${}^4C_2\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^2$	${}^4C_3\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)$	${}^4C_4\left(\frac{1}{5}\right)^4$
		$= \frac{256}{625}$	$= \frac{96}{625}$	$= \frac{16}{625}$	$= \frac{1}{625}$

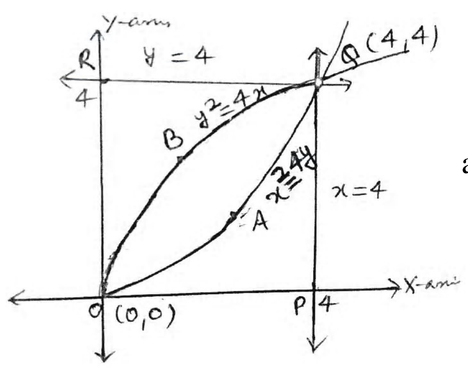
$2\frac{1}{2}$

Mean (μ) = $\Sigma X.P(X) = 0\times\frac{256}{625}+1\times\frac{256}{625}+2\times\frac{96}{625}+3\times\frac{16}{625}+4\times\frac{1}{625}=\frac{4}{5}$ 1

Variance (σ^2) $= \Sigma x^2.P(x) - [\Sigma x.P(x)]^2$

$$= 0\times\frac{256}{625}+\frac{1\times 256}{625}+\frac{4\times 96}{625}+\frac{9\times 16}{625}+\frac{16}{625}-\left(\frac{4}{5}\right)^2=\frac{16}{25}$$ 1

24. Point of intersection of $y^2 = 4x$ and $x^2 = 4y$ are (0, 0) and (4, 4);



Correct Graph $1\frac{1}{2}$

$$\text{are (OAQBO)} = \int_0^4 \left(2\sqrt{x}-\frac{x^2}{4}\right)dx$$ 1

$$= \left\{\frac{4}{3}x^{3/2}-\frac{x^3}{12}\right\}\Bigg]_0^4$$

$$= \frac{32}{3}-\frac{16}{3}=\frac{16}{3}$$ $\frac{1}{2}$

$$\text{area (OPQAO)} = \int_0^4 \frac{x^2}{4}dx = \frac{1}{12}x^3\Bigg]_0^4 = \frac{16}{3}$$ $1\frac{1}{2}$

$$\text{area (OBQRO)} = \int_0^4 \frac{y^2}{4}dy = \frac{1}{12}y^3\Bigg]_0^4 = \frac{16}{3}$$ $1\frac{1}{2}$

Hence the areas of the three regions are equal.

25. Commutative: For any elements a, b $\in$ A

a * b = a + b + ab = b + a + ba = b * a. Hence * is commutative $1\frac{1}{2}$

Associative: For any three elements a, b, c, $\in$ A

a * (b * c) = a * (b + c + bc) = a + b + c + bc + ab + ac + abc
(a * b) * c = (a + b + ab) * c = a + b + ab + c + ac + bc + abc $1\frac{1}{2}$

$\therefore$ a * (b * c) = (a * b) * c, Hence * is Associative.

Identity element: let e $\in$ A be the identity element them a * e = e * a = a

$\Rightarrow$ a + e + ae = e + a + ea = a $\Rightarrow$ e (1 + a) = 0, as $a \neq -1$

e = 0 is the identity element $1\frac{1}{2}$

Invertible: let a, b $\in$ A so that 'b' is inverse of a

$\therefore$ a * b = b * a = e

$\Rightarrow$ a + b + ab = b + a + ba = 0

As $a \neq -1$, $b = \frac{-a}{1+a} \in A$. Hence every element of A is invertible $1\frac{1}{2}$

26. Line through 'P' and perpendicular to plane is:

$\vec{r} = (2\hat{i} + 3\hat{j} + 4\hat{k}) + \lambda(2\hat{i} + \hat{j} + 3\hat{k})$ 1

General point on line is: $\vec{r} = (2 + 2\lambda)\hat{i} + (3+\lambda)\hat{j} + (4+3\lambda)\hat{k}$

For some $\lambda \in R$, $\vec{r}$ is the foot of perpendicular, say Q, from P to the plane, since it lies on plane

$\therefore \quad [(2 + 2\lambda)\hat{i} + (3 + \lambda)\hat{j} + (4 + 3\lambda)\hat{k}]\cdot(2\hat{i} + \hat{j} + 3\hat{k}) - 26 = 0$

$\Rightarrow \quad 4 + 4\lambda + 3 + \lambda + 12 + 9\lambda - 26 = 0 \;\Rightarrow\; \lambda = \frac{1}{2}$ $1\frac{1}{2}$

$\therefore$ Foot of perpendicular is $Q\left(3\hat{i} + \frac{7}{2}\hat{j} + \frac{11}{2}\hat{k}\right)$ 1

let $P'(a\hat{i} + b\hat{j} + c\hat{k})$ be the image of P in the plane then Q is mid point of PP′

$\therefore \quad Q\left(\frac{a+2}{2}\hat{i} + \frac{b+3}{2}\hat{j} + \frac{c+4}{2}\hat{k}\right) = Q\left(3\hat{i} + \frac{7}{2}\hat{j} + \frac{11}{2}\hat{k}\right)$ 1

$\Rightarrow \quad \frac{a+2}{2} = 3, \frac{b+3}{2} = \frac{7}{2}, \frac{c+4}{2} = \frac{11}{2} \;\Rightarrow\; a = 4, b = 4, c = 7 \;\; \therefore P'(4\hat{i} + 4\hat{j} + 7\hat{k})$ 1

Perpendicular distance of P from plane = $PQ = \sqrt{(2-3)^2 + \left(3 - \frac{7}{2}\right)^2 + \left(4 - \frac{11}{2}\right)^2} = \sqrt{\frac{7}{2}}$ $\frac{1}{2}$

QUESTION PAPER CODE 65/3/C

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $\dfrac{2\,(2\vec{a}+3\vec{b})+1\,(3\vec{a}-2\vec{b})}{2+1}$ $\dfrac{1}{2}$

$= \dfrac{7}{3}\vec{a}+\dfrac{4}{3}\vec{b}$ (or enternal division may also be considered) $\dfrac{1}{2}$

2. 2 1

3. $\dfrac{x}{3}+\dfrac{y}{-4}+\dfrac{z}{2} = 1$ $\dfrac{1}{2}$

$\Rightarrow\ \vec{r}\cdot(4\hat{i}-3\hat{j}+6\hat{k}) = 12$ or $\vec{r}\cdot\left(\dfrac{\hat{i}}{3}-\dfrac{\hat{j}}{4}+\dfrac{\hat{k}}{2}\right)=1$ $\dfrac{1}{2}$

4. $(x + 3)2x - (-2)\,(-3x) = 8$ $\dfrac{1}{2}$

$x = 2$ $\dfrac{1}{2}$

5. $\begin{pmatrix} 2 & 5 \\ 2 & 4 \end{pmatrix} = \begin{pmatrix} 3 & 1 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} 1 & 2 \\ -1 & -1 \end{pmatrix}$ $\dfrac{1}{2}+\dfrac{1}{2}$

6. $\left.\begin{array}{l}\text{No. of possible matrices} = 3^4 \\ \qquad\qquad\qquad\text{or}\quad 81\end{array}\right\}$ 1

SECTION B

7. Let $I = \displaystyle\int_0^{\pi/2} \frac{\sin^2 x}{\sin x+\cos x}\,dx$, Also $I = \displaystyle\int_0^{\pi/2} \frac{\sin^2\left(\frac{\pi}{2}-x\right)}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}\,dx = \int_0^{\pi/2}\frac{\cos^2 x}{\cos x+\sin x}\,dx$ 1

Adding to get, $2I = \displaystyle\int_0^{\pi/2}\frac{1}{\sin x+\cos x}\,dx = \frac{1}{\sqrt{2}}\int_0^{\pi/2}\frac{1}{\cos\left(x-\pi/4\right)}\,dx$ $\dfrac{1}{2}+1$

$\Rightarrow 2I = \dfrac{1}{\sqrt{2}}\displaystyle\int_0^{\pi/2}\sec\left(x-\pi/4\right)dx = \frac{1}{\sqrt{2}}\log\left|\sec\left(x-\frac{\pi}{4}\right)+\tan\left(x-\frac{\pi}{4}\right)\right|_0^{\pi/2}$ 1

$\Rightarrow 2I = \dfrac{1}{\sqrt{2}}\left\{\log\left|\sqrt{2}+1\right|-\log\left|\sqrt{2}-1\right|\right\}$

$\Rightarrow I = \dfrac{1}{2\sqrt{2}}\left\{\log\left|\sqrt{2}+1\right|-\log\left|\sqrt{2}-1\right|\right.$ or $\dfrac{1}{2\sqrt{2}}\log\left|\dfrac{\sqrt{2}+1}{\sqrt{2}-1}\right|$ $\dfrac{1}{2}$

OR

$\displaystyle\int_0^{3/2}|x\cos\pi x|\,dx = \int_0^{1/2}x\cos\pi x\,dx - \int_{1/2}^{3/2}x\cos\pi x\,dx$ $1\dfrac{1}{2}$

$$= \left\{\frac{x\sin\pi x}{\pi} + \frac{\cos\pi x}{\pi^2}\right\}_0^{1/2} - \left\{\frac{x\sin\pi x}{\pi} + \frac{\cos\pi x}{\pi^2}\right\}_{1/2}^{3/2}$$ $1\frac{1}{2}$

$$= \frac{1}{2\pi} - \frac{1}{\pi^2} - \left(-\frac{3}{2\pi} - \frac{1}{2\pi}\right) = \frac{5}{2\pi} - \frac{1}{\pi^2}$$ 1

8. Let X = Amount he wins then x = ₹ 5, 4, 3, – 3 1

P = Probability of getting a no. >4 = $\frac{1}{3}$, q = $1 - p = \frac{2}{3}$ $\frac{1}{2}$

X:	5	4	3	–3
P(x)	$\frac{1}{3}$	$\frac{2}{3}\cdot\frac{1}{3} = \frac{2}{9}$	$\left(\frac{2}{3}\right)^2\cdot\frac{1}{3} = \frac{4}{27}$	$\left(\frac{2}{3}\right)^3 = \frac{8}{27}$

2

Expected amount he wins $= \Sigma\, XP(X) = \frac{5}{3} + \frac{8}{9} + \frac{12}{27} - \frac{24}{27}$

$$= ₹\,\frac{19}{9} \text{ or } ₹\,2\frac{1}{9}$$ $\frac{1}{2}$

OR

E_1 = Event that all balls are white,
E_2 = Event that 3 balls are white and 1 ball is non white
E_3 = Event that 2 balls are white and 2 balls are non-white
A = Event that 2 balls drawn without replacement are white

1

$P(E_1) = P(E_2) = P(E_3) = \frac{1}{3}$ $\frac{1}{2}$

$P(A/E_1) = 1$, $P(A/E_2) = \frac{3}{4}\cdot\frac{2}{3} = \frac{1}{2}$, $P(A/E_3) = \frac{2}{4}\cdot\frac{1}{3} = \frac{1}{6}$ $1\frac{1}{2}$

$$P(E_1/A) = \frac{1.\frac{1}{3}}{1\cdot\frac{1}{3} + \frac{1}{3}\cdot\frac{1}{2} + \frac{1}{3}\cdot\frac{1}{6}} = \frac{3}{5}$$ 1

9. Let $x^2 = t$ $\therefore$ $\frac{x^2}{x^4 + x^2 - 2} = \frac{x^2}{(x^2-1)(x^2+2)} = \frac{t}{(t-1)(t+2)} = \frac{A}{t-1} + \frac{B}{t+2}$ 1

Solving for A and B to get, $A = \frac{1}{3}, B = \frac{2}{3}$ 1

$$\int \frac{x^2}{x^4 + x^2 - 2}\,dx = \frac{1}{3}\int\frac{1}{x^2-1}\,dx + \frac{2}{3}\int\frac{1}{x^2+2}\,dx = \frac{1}{6}\log\left|\frac{x-1}{x+1}\right| + \frac{\sqrt{2}}{3}\tan^{-1}\frac{x}{\sqrt{2}} + C$$ 1 + 1

10. $\frac{dx}{dt}$ = 2a cos 2t (1 + cos 2t) – 2a sin 2t·sin 2t $1\frac{1}{2}$

$\frac{dy}{dt}$ = –2b sin 2t (1 – cos 2t) + 2b cos 2t·sin 2t 1

$$\left.\frac{dy}{dx}\right]_{t=\frac{\pi}{4}} = \left.\frac{2b\cos 2t\cdot\sin 2t - 2b\sin 2t\,(1-\cos 2t)}{2a\cos 2t\,(1+\cos 2t) - 2a\sin 2t\cdot\sin 2t}\right]_{t=\frac{\pi}{4}} = \frac{b}{a}$$ $\frac{1}{2}$ + 1

11. Equation of line through A(3, 4, 1) and B(5, 1, 6)

$$\frac{x-3}{2} = \frac{y-4}{-3} = \frac{z-1}{5} = k(\text{say})$$ 1

General point on the line:

$x = 2k + 3,\ y = -3k + 4,\ z = 5k + 1$ $\frac{1}{2}$

line crosses xz plane i.e. $y = 0$ if $-3k + 4 = 0$

$\therefore \quad k = \frac{4}{3}$ 1

Co-ordinate of required point $\left(\frac{17}{3}, 0, \frac{23}{3}\right)$ $\frac{1}{2}$

Angle, which line makes with xz plane:

$$\sin\theta = \left|\frac{2(0)+(-3)(1)+5(0)}{\sqrt{4+9+25}\sqrt{1}}\right| = \frac{3}{\sqrt{38}} \Rightarrow \theta = \sin^{-1}\left(\frac{3}{\sqrt{38}}\right)$$ 1

12. $$\int (3x+1)\sqrt{4-3x-2x^2}\,dx = -\frac{3}{4}\int(-4x-3)\sqrt{4-3x-2x^2}\,dx - \frac{5}{4}\int\sqrt{4-3x-2x^2}\,dx$$ 1

$$= -\frac{1}{2}(4-3x-2x^2)^{3/2} - \frac{5}{4}\sqrt{2}\int\sqrt{\left(\frac{\sqrt{41}}{4}\right)^2 - \left(x+\frac{3}{4}\right)^2}\,dx$$ 1 + 1

$$\left.\begin{aligned} &= -\frac{1}{2}(4-3-2x^2)^{3/2} - \frac{5}{4}\sqrt{2}\left\{\frac{4x+3}{8}\sqrt{\frac{41}{16} - \left(x+\frac{3}{4}\right)^2} + \frac{41}{32}\cdot\sin^{-1}\left(\frac{4x+3}{\sqrt{41}}\right)\right\} + C \\ &= -\frac{1}{2}(4-3x-2x^2)^{3/2} - \frac{5}{4}\left\{\frac{4x+3}{8}\sqrt{4-3x-2x^2} + \frac{41\sqrt{2}}{32}\cdot\sin^{-1}\left(\frac{4x+3}{\sqrt{41}}\right)\right\} + C \end{aligned}\right\}$$ 1

13. $y^2 = ax^3 + b \Rightarrow 2y\frac{dy}{dx} = 3ax^2 \therefore \frac{dy}{dx} = \frac{3a}{2}\frac{x^2}{y}$ 1

Slope of tangent at (2, 3) = $\left.\frac{dy}{dx}\right]_{(2,\,3)} = \frac{3a}{2}\cdot\frac{4}{3} = 2a$ 1

Comparing with slope of tangent $y = 4x - 5$, we get, $2a = 4 \therefore \boxed{a = 2}$ 1

Also (2, 3) lies on the curve $\therefore 9 = 8a + b$, put $a = 2$, we get $b = -7$ 1

14. Let ₹ x be invested in first bond

and ₹ y be invested in second bond

then the system of equations is:

$$\left.\begin{aligned}\frac{10x}{100} + \frac{12y}{100} = 2800 \\ \frac{12x}{100} + \frac{10y}{100} = 2700\end{aligned}\right\} \Rightarrow \left.\begin{aligned}5x + 6y = 140000 \\ 6x + 5y = 135000\end{aligned}\right\}$$ 1

let $A = \begin{bmatrix} 5 & 6 \\ 6 & 5 \end{bmatrix};\ X = \begin{bmatrix} x \\ y \end{bmatrix};\ B = \begin{bmatrix} 140000 \\ 135000 \end{bmatrix}$

$\therefore\ A \cdot X = B$

$|A| = -11;\ A^{-1} = \dfrac{1}{-11}\begin{bmatrix} 5 & -6 \\ -6 & 5 \end{bmatrix}$ 1

$$\left.\begin{array}{l}\therefore \text{ Solution is } X = A^{-1}B \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \dfrac{1}{-11}\begin{bmatrix} 5 & -6 \\ -6 & 5 \end{bmatrix}\begin{bmatrix} 140000 \\ 135000 \end{bmatrix} = \begin{bmatrix} 10000 \\ 15000 \end{bmatrix} \\ \therefore x = 10000,\ y = 15000, \therefore \text{ Amount invested} = ₹\ 25000 \end{array}\right\}$$ $\frac{1}{2}+\frac{1}{2}$

Value: caring elders 1

15. The differential equation can be re-written as:

$\dfrac{dy}{dx} = \dfrac{x-y}{x+y}$, put $y = vx$, $\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$ $\frac{1}{2} + \frac{1}{2}$

$\Rightarrow\ v + x\dfrac{dv}{dx} = \dfrac{1-v}{1+v} \Rightarrow \dfrac{1+v}{1-2v-v^2}dv = \dfrac{1}{x}dx$ 1

integrating we get

$\Rightarrow\ \dfrac{1}{2}\displaystyle\int \dfrac{2V+2}{V^2+2V-1}dv = -\int \dfrac{1}{x}dx = \dfrac{1}{2}\log|V^2+2V-1| = -\log x + \log C$ $1\frac{1}{2}$

$\therefore$ Solution of the differential equation is:

$\dfrac{1}{2}\log\left|\dfrac{y^2}{x^2} + \dfrac{2y}{x} - 1\right| = \log C - \log x$ or, $y^2 + 2xy - x^2 = C^2$ $\frac{1}{2}$

16. let $\vec{d}_1$ & $\vec{d}_2$ be the two diagonal vectors:

$\therefore\quad \vec{d}_1 = 4\hat{i} - 2\hat{j} - 2\hat{k},\ \vec{d}_2 = -6\hat{j} - 8\hat{k}$ $\frac{1}{2}+\frac{1}{2}$

or $\quad \vec{d}_2 = 6\hat{j} + 8\hat{k}$

Unit vectors parallel to the diagonals are:

$\hat{d}_1 = \dfrac{2}{\sqrt{6}}\hat{i} - \dfrac{1}{\sqrt{6}}\hat{j} - \dfrac{1}{\sqrt{6}}\hat{k}$ $\frac{1}{2}$

$\hat{d}_2 = -\dfrac{3}{5}\hat{j} - \dfrac{4}{5}\hat{k}\quad \left(\text{or } \hat{d}_2 = \dfrac{3}{5}\hat{j} + \dfrac{4}{5}\hat{k}\right)$ $\frac{1}{2}$

$\vec{d}_1 \times \vec{d}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 4 & -2 & -2 \\ 0 & -6 & -8 \end{vmatrix} = 4\hat{i} + 32\hat{j} - 24\hat{k}$ 1

Area of parallelogram $= \dfrac{1}{2}\left|\vec{d}_1 \times \vec{d}_2\right| = \sqrt{404}$ or $2\sqrt{101}$ sq. units 1

17. $\sin^{-1}x + \sin^{-1}(1-x) = \cos^{-1}x \ \Rightarrow\ \sin^{-1}(1-x) = \dfrac{\pi}{2} - 2\sin^{-1}x$ 1

$\Rightarrow\ 1 - x = \sin\left(\dfrac{\pi}{2} - 2\sin^{-1}x\right) \Rightarrow 1 - x = \cos(2\sin^{-1}x) \Rightarrow 1 - x = 1 - 2\sin^2(\sin^{-1}x)$ 1

$\Rightarrow\ 1 - x = 1 - 2x^2$ 1

Solving we get, $x = 0$ or $x = \dfrac{1}{2}$ 1

OR

From the equation: $\cos^{-1}\dfrac{x}{a} = \alpha - \cos^{-1}\dfrac{y}{b}$

$$\frac{x}{a} = \cos\left(\alpha - \cos^{-1}\frac{y}{b}\right) \Rightarrow \frac{x}{a} = \cos\alpha\cdot\cos\left(\cos^{-1}\frac{y}{b}\right) + \sin\alpha\cdot\sin\left(\cos^{-1}\frac{y}{b}\right)$$ 1 + 1

$$\Rightarrow \frac{x}{a} = \frac{y\cdot\cos\alpha}{b} + \sin\alpha\sqrt{1-\frac{y^2}{b^2}} \Rightarrow \frac{x}{a} - \frac{y}{b}\cos\alpha = \sin\alpha\sqrt{1-\frac{y^2}{b^2}}$$ 1

Squaring both sides,

$$\Rightarrow \left(\frac{x}{a} - y\frac{\cos\alpha}{b}\right)^2 = \left(\sin\alpha\sqrt{1-\frac{y^2}{b^2}}\right)^2$$ $\frac{1}{2}$

$$\Rightarrow \frac{x^2}{a^2} - \frac{2xy}{ab}\cdot\cos\alpha + \frac{y^2}{b^2} = \sin^2\alpha.$$ $\frac{1}{2}$

18. Let $y = u + v,\ u = x^{\sin x},\ v = (\sin x)^{\cos x}$

$$\log u = \sin x.\log x \Rightarrow \frac{du}{dx} = x^{\sin x}\cdot\left\{\cos x.\log x + \frac{\sin x}{x}\right\}$$ $\frac{1}{2}+1$

$$\log v = \cos x.\log(\sin x) \Rightarrow \frac{dv}{dx} = (\sin x)^{\cos x}\cdot\{\cos x\cdot\cot x - \sin x\cdot\log(\sin x)\}$$ $\frac{1}{2}+1$

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} = x^{\sin x}\cdot\left\{\cos x\cdot\log x + \frac{\sin x}{x}\right\} + (\sin x)^{\cos x}\{\cos x\cdot\cot x - \sin x\cdot\log(\sin x)\}$$ $\frac{1}{2}+\frac{1}{2}$

OR

$$\frac{dy}{dx} = \frac{-2\sin(\log x)}{x} + \frac{3\cos(\log x)}{x}$$ 1

$$\Rightarrow x\frac{dy}{dx} = -2\sin(\log x) + 3\cos(\log x),\text{ differentiate w.r.t 'x'}$$ $\frac{1}{2}$

$$\Rightarrow x\frac{d^2y}{dx^2} + \frac{dy}{dx} = \frac{-2\cos(\log x)}{x} - \frac{3\sin(\log x)}{x}$$ 2

$$\Rightarrow x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} = -y \Rightarrow x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$$ $\frac{1}{2}$

19. Let radius of any of the circle touching co-ordinate axes in the second quadrant be "a" then centre is (–a, a)

$\therefore$ Equation of the family of circles is:

$$(x+a)^2 + (y-a)^2 = a^2,\ a \in R$$ $1\frac{1}{2}$

$$\Rightarrow x^2 + y^2 + 2ax - 2ay + a^2 = 0$$

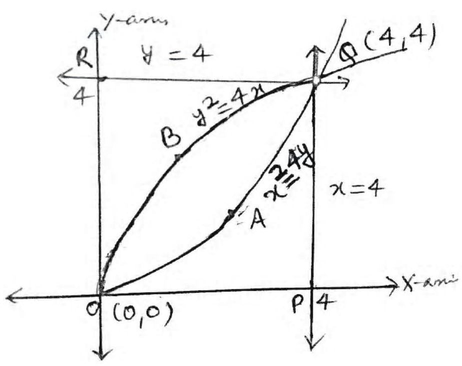
Differentiate w.r.t. "x", $2x + 2yy' + 2a - 2ay' = 0 \Rightarrow a = \dfrac{x + yy'}{y' - 1}$ $1\frac{1}{2}$

$\therefore$ The differential equation is:

$$\left.\begin{aligned}&\left(x+\frac{x+yy'}{y'-1}\right)^2+\left(y-\frac{x+yy'}{y'-1}\right)^2=\left(\frac{x+yy'}{y'-1}\right)^2\\ &\Rightarrow\left(\frac{xy'+yy'}{y'-1}\right)^2+\left(\frac{x+y}{y'-1}\right)^2=\left(\frac{x+yy'}{y'-1}\right)^2\end{aligned}\right\}$$ 1

SECTION C

20. Point of intersection of $y^2 = 4x$ and $x^2 = 4y$ are (0, 0) and (4, 4);



Correct Graph $1\frac{1}{2}$

$$\text{are (OAQBO)} = \int_0^4 \left(2\sqrt{x} - \frac{x^2}{4}\right) dx$$ 1

$$= \left\{\frac{4}{3}x^{3/2} - \frac{x^3}{12}\right\}\Bigg]_0^4$$

$$= \frac{32}{3} - \frac{16}{3} = \frac{16}{3}$$ $\frac{1}{2}$

$$\text{area (OPQAO)} = \int_0^4 \frac{x^2}{4} dx = \frac{1}{12}x^3\Bigg]_0^4 = \frac{16}{3}$$ $1\frac{1}{2}$

$$\text{area (OBQRO)} = \int_0^4 \frac{y^2}{4} dy = \frac{1}{12}y^3\Bigg]_0^4 = \frac{16}{3}$$ $1\frac{1}{2}$

Hence the areas of the three regions are equal.

21. Commutative: For any elements a, b $\in$ A

a * b = a + b + ab = b + a + ba = b * a. Hence * is commutative $1\frac{1}{2}$

Associative: For any three elements a, b, c, $\in$ A

a * (b * c) = a * (b + c + bc) = a + b + c + bc + ab + ac + abc

(a * b) * c = (a + b + ab) * c = a + b + ab + c + ac + bc + abc $1\frac{1}{2}$

$\therefore$ a * (b * c) = (a * b) * c, Hence * is Associative.

Identity element: let e $\in$ A be the identity element them a * e = e * a = a

$\Rightarrow$ a + e + ae = e + a + ea = a $\Rightarrow$ e (1 + a) = 0, as a $\neq$ – 1

e = 0 is the identity element $1\frac{1}{2}$

Invertible: let a, b $\in$ A so that 'b' is inverse of a

$\therefore$ a * b = b * a = e

$\Rightarrow$ a + b + ab = b + a + ba = 0

As a $\neq$ –1, b = $\frac{-a}{1+a}$ $\in$ A. Hence every element of A is invertible $1\frac{1}{2}$

22. $$\begin{vmatrix} 1 & 1 & 1 \\ 1+\cos A & 1+\cos B & 1+\cos C \\ \cos^2 A+\cos A & \cos^2 B+\cos B & \cos^2 C+\cos C \end{vmatrix}=0$$

Apply $C_2 \to C_2 - C_1$, $C_3 \to C_3 - C_1$

$$\Leftrightarrow \begin{vmatrix} 1 & 0 & 0 \\ 1+\cos A & \cos B-\cos A & \cos C-\cos A \\ \cos^2 A+\cos A & (\cos B-\cos A)(\cos B+\cos A+1) & (\cos C-\cos A)(\cos C+\cos A+1) \end{vmatrix}=0$$ 3

Taking (cos B – cos A), (cos C – cos A) common from C_2 & C_3

$$\Leftrightarrow (\cos B-\cos A)(\cos C-\cos A)\begin{vmatrix} 1 & 0 & 0 \\ 1+\cos A & 1 & 1 \\ \cos^2 A+\cos A & \cos B+\cos A+1 & \cos C+\cos A+1 \end{vmatrix}=0$$ 1

Expand along R_1

$\Leftrightarrow$ (cos B – cos A) (cos C – cos A) (cos C – cos B) = 0 1

$\Leftrightarrow$ cos A = cos B $\quad\Leftrightarrow$ A = B $\quad\Leftrightarrow$ ΔABC is an isosceles triangle 1

or $\quad$ or

cos B = cos C $\quad$ B = C

or $\quad$ or

cos C = cos A $\quad$ C = A

OR

let the cost of one pen of veriety 'A', 'B' and 'C' be ₹ x. ₹ y and ₹ z respectively then the system of equations is:

$$\left.\begin{aligned} x+y+z&=21 \\ 4x+3y+2z&=60 \\ 6x+2y+3z&=70 \end{aligned}\right\}$$ $1\frac{1}{2}$

Matrix form of the system is:

$$A\cdot X = B, \text{ where } A=\begin{bmatrix} 1 & 1 & 1 \\ 4 & 3 & 2 \\ 6 & 2 & 3 \end{bmatrix}; X=\begin{bmatrix} x \\ y \\ z \end{bmatrix}; B=\begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix}$$ $\frac{1}{2}$

|A| = (5) –1 (0) + 1 (–10) = – 5 1

co-factors of the matrix A are:

$$\left.\begin{matrix} C_{11}=5; & C_{21}=-1; & C_{31}=-1 \\ C_{12}=0; & C_{22}=-3 & C_{32}=2 \\ C_{13}=-10; & C_{23}=4; & C_{33}=-1 \end{matrix}\right\}$$ 2

$$\therefore\ A^{-1} = \frac{1}{|A|}\text{Adj } A=\frac{1}{-5}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}$$ $\frac{1}{2}$

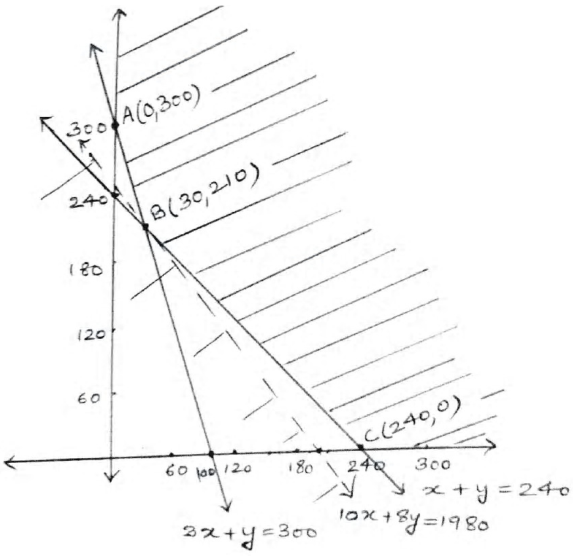
Solution of the matrix equation is $X = A^{-1} B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{5}\begin{bmatrix} 5 & -1 & -1 \\ 0 & -3 & 2 \\ -10 & 4 & -1 \end{bmatrix}\begin{bmatrix} 21 \\ 60 \\ 70 \end{bmatrix}=\begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix} \therefore x=5, y=8, z=8$$ $\frac{1}{2}$

23. Let x kg of fertilizer A be used

and y kg of fertilizer B be used

then the linear programming problem is:



Minimise cost: z = 10x + 8y 1

$$\left.\begin{aligned}&\text{Subject to } \frac{12x}{100}+\frac{4y}{100}\ge 12 \Rightarrow 3x+y\ge 300\\ &\frac{5x}{100}+\frac{5y}{100}\ge 12 \Rightarrow x+y\ge 240\\ &x, y\ge 0\end{aligned}\right\}$$ 2

Correct Graph $1\frac{1}{2}$

Value of Z at corners of the unbounded region ABC:

Corner	Value of Z
A (0, 300)	₹ 2400
B(30, 210)	₹ 1980 (Minimum)
C(240, 0)	₹ 2400

1

The region of 10x + 8y < 1980 or 5x + 4y < 990 has no point in common to the feasible region. Hence, minimum cost = ₹ 1980 at x = 30 and y = 210 $\frac{1}{2}$

24. Line through 'P' and perpendicular to plane is:

$\vec{r} = (2\hat{i}+3\hat{j}+4\hat{k}) + \lambda(2\hat{i}+\hat{j}+3\hat{k})$ 1

General point on line is: $\vec{r} = (2+2\lambda)\hat{i} + (3+\lambda)\hat{j} + (4+3\lambda)\hat{k}$

For some $\lambda \in R$, $\vec{r}$ is the foot of perpendicular, say Q, from P to the plane, since it lies on plane

$\therefore \quad [(2+2\lambda)\hat{i} + (3+\lambda)\hat{j} + (4+3\lambda)\hat{k}]\cdot(2\hat{i}+\hat{j}+3\hat{k}) - 26 = 0$

$\Rightarrow \quad 4+4\lambda+3+\lambda+12+9\lambda-26=0 \;\Rightarrow\; \lambda = \frac{1}{2}$ $1\frac{1}{2}$

$\therefore$ Foot of perpendicular is $Q\left(3\hat{i}+\frac{7}{2}\hat{j}+\frac{11}{2}\hat{k}\right)$ 1

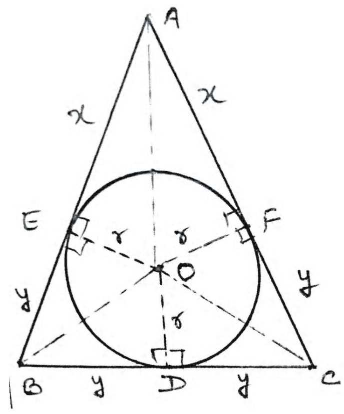
let $P'(a\hat{i}+b\hat{j}+c\hat{k})$ be the image of P in the plane then Q is mid point of PP′

$\therefore \quad Q\left(\frac{a+2}{2}\hat{i}+\frac{b+3}{2}\hat{j}+\frac{c+4}{2}\hat{k}\right) = Q\left(3\hat{i}+\frac{7}{2}\hat{j}+\frac{11}{2}\hat{k}\right)$ 1

$\Rightarrow \quad \frac{a+2}{2}=3, \frac{b+3}{2}=\frac{7}{2}, \frac{c+4}{2}=\frac{11}{2} \;\Rightarrow\; a=4, b=4, c=7 \;\; \therefore P'(4\hat{i}+4\hat{j}+7\hat{k})$ 1

Perpendicular distance of P from plane = $PQ = \sqrt{(2-3)^2+\left(3-\frac{7}{2}\right)^2+\left(4-\frac{11}{2}\right)^2} = \sqrt{\frac{7}{2}}$ $\frac{1}{2}$

25. Correct Figure 1



Let ΔABC be isosceles with inscribed circle of radius 'r' touching sides AB, AC and BC at E, F and D respectively.

let AE = AF = x, BE = BD = y, CF = CD = y then

area (ΔABC) = ar(ΔAOB) + ar(ΔAOC) + ar(ΔBOC)

$$\Rightarrow \frac{1}{2}.2y\left(r+\sqrt{r^2+x^2}\right) = \frac{1}{2}\{2yr+2(x+y)r\} \Rightarrow x = \frac{2r^2y}{y^2-r^2}$$ 1

Then,

$$\text{P(Perimeter of } \Delta ABC) = 2x + 4y = \frac{4r^2y}{y^2-r^2} + 4y$$ 1

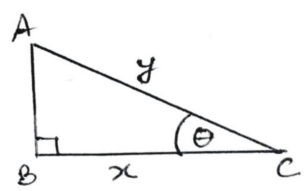
$$\frac{dP}{dy} = \frac{-4r^2(r^2+y^2)}{(y^2-r^2)^2} + 4 \text{ and } \frac{dP}{dy} = 0 \Rightarrow y = \sqrt{3}r$$ $1+\frac{1}{2}$

$$\left.\frac{d^2P}{dy^2}\right]_{y=\sqrt{3}r} = \frac{4r^2y(2y^2+6r^2)}{(y^2-r^2)^3} = \frac{6\sqrt{3}}{r} > 0$$ $\frac{1}{2}$

∴ Perimeter is least iff $y = \sqrt{3}r$ and least perimeter is

$$P = 4y + \frac{4r^2y}{y^2-r^2} = 4\sqrt{3}r + \frac{4r^2\sqrt{3}r}{2r^2} = 6\sqrt{3}r$$ 1

OR



let ABC be the right triangle with ∠B = 90°

∠ACB = θ, AC = y, BC = x, x + y = k (constant)

$$\text{A (Area of triange)} = \frac{1}{2}.\text{BC. AB} = \frac{1}{2}.x\sqrt{y^2-x^2}$$ $1\frac{1}{2}$

$$\text{let } z = A^2 = \frac{1}{4}x^2(y^2-x^2) = \frac{1}{4}x^2\{(k-x)^2-x^2\} = \frac{1}{4}(x^2k^2-2kx^3)$$ 1

$$\frac{dz}{dx} = \frac{1}{4}(2xk^2-6kx^2) \text{ and } \frac{dz}{dx} = 0 \Rightarrow x = \frac{k}{3}, y = k - x = \frac{2k}{3}$$ 1+1

$$\left.\frac{d^2z}{dx^2}\right]_{x=\frac{k}{3}} = \left.\frac{1}{4}(2k^2-12\,kx)\right]_{x=\frac{k}{3}} = -\frac{k^2}{2} < 0$$ $\frac{1}{2}$

∴ z and area of ΔABC is max at $x = \frac{k}{3}$

$$\text{and, } \cos\theta = \frac{x}{y} = \frac{k}{3}.\frac{3}{2k} = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$$ 1

26. Let X = Number of bad oranges out of 4 drawn = 0, 1, 2, 3, 4 1

P = Probability of a bad orange = $\frac{1}{5}$, $q = 1 - p = \frac{4}{5}$ $\frac{1}{2}$

$\therefore$ Probability distribution is:

X:	0	1	2	3	4
P(X):	${}^4C_0\left(\frac{4}{5}\right)^4 = \frac{256}{625}$	${}^4C_1 \frac{1}{5}\left(\frac{4}{5}\right)^3$	${}^4C_2\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)^2$	${}^4C_3\left(\frac{1}{5}\right)^2\left(\frac{4}{5}\right)$	${}^4C_4\left(\frac{1}{5}\right)^4$
		$= \frac{256}{625}$	$= \frac{96}{625}$	$= \frac{16}{625}$	$= \frac{1}{625}$

$2\frac{1}{2}$

Mean (μ) = $\Sigma X.P(X) = 0\times\frac{256}{625} + 1\times\frac{256}{625} + 2\times\frac{96}{625} + 3\times\frac{16}{625} + 4\times\frac{1}{625} = \frac{4}{5}$ 1

Variance (σ^2) $= \Sigma x^2.P(x) - [\Sigma x.P(x)]^2$

$$= 0\times\frac{256}{625} + \frac{1\times 256}{625} + \frac{4\times 96}{625} + \frac{9\times 16}{625} + \frac{16}{625} - \left(\frac{4}{5}\right)^2 = \frac{16}{25}$$ 1