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Senior School Certificate Examination

March 2016

Marking Scheme — Mathematics 65/1/E, 65/2/E, 65/3/E

General Instructions:

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggested answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage

2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. In question (s) on differential equations, constant of integration has to be written.

5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.

6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.

7. Separate Marking Scheme for all the three sets has been given.

8. As per orders of the Hon'ble Supreme Court. The candidates would now be permitted to obtain photocopy of the Answer book on request on payment of the prescribed fee. All examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

QUESTION PAPER CODE 65/1/E

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $R_1 \to R_1 + R_2 + R_3$ or $C_1 \to C_1 + C_2 + C_3$ $\frac{1}{2}$

Ans. 0 $\frac{1}{2}$

2. $b_{21} = -16,\ b_{23} = -2$ [For any one correct value] $\frac{1}{2}$

$b_{21} + b_{23} = -16 + (-2) = -18$ $\frac{1}{2}$

3. 2^6 or 64 1

4. $(\alpha, -\beta, \gamma)$ 1

5. $\frac{1(\vec{a}-\vec{b})+3(\vec{a}+3\vec{b})}{4}$ (i.e., using correct formula) $\frac{1}{2}$

$= \vec{a} + 2\vec{b}$ $\frac{1}{2}$

6. Finding $\cos\theta = \frac{\sqrt{3}}{2}$ $\frac{1}{2}$

$|\vec{a} \times \vec{b}| = 6$ $\frac{1}{2}$

SECTION B

7. $\tan^{-1}\left(\frac{2-x}{2+x}\right) = \frac{1}{2}\tan^{-1}\frac{x}{2}$

$\Rightarrow \quad 2\tan^{-1}\left(\frac{2-x}{2+x}\right) = \tan^{-1}\frac{x}{2}$ $\frac{1}{2}$

$\Rightarrow \quad \tan^{-1}\frac{2\left(\frac{2-x}{2+x}\right)}{1-\left(\frac{2-x}{2+x}\right)^2} = \tan^{-1}\frac{x}{2}$ $1\frac{1}{2}$

$\Rightarrow \quad \tan^{-1}\frac{4-x^2}{4x} = \tan^{-1}\frac{x}{2}$ 1

$\Rightarrow \quad \frac{4-x^2}{4x} = \frac{x}{2}$ $\frac{1}{2}$

$\Rightarrow \quad x = \frac{2}{\sqrt{3}}$ $(\because x > 0)$ $\frac{1}{2}$

OR

$$2\sin^{-1}\left(\frac{3}{5}\right)-\tan^{-1}\left(\frac{17}{31}\right)$$

$$=2\tan^{-1}\left(\frac{3}{4}\right)-\tan^{-1}\left(\frac{17}{31}\right)$$ 1

$$=\tan^{-1}\frac{2\times\frac{3}{4}}{1-\left(\frac{3}{4}\right)^2}-\tan^{-1}\left(\frac{17}{31}\right)$$ 1

$$=\tan^{-1}\frac{24}{7}-\tan^{-1}\frac{17}{31}$$

$$=\tan^{-1}\left(\frac{\frac{24}{7}-\frac{17}{31}}{1+\frac{24}{7}\times\frac{17}{31}}\right)$$ 1

$$=\tan^{-1}(1)$$ 1

$$=\frac{\pi}{4}$$

8. Let the number of children be x and the amount distributed by Seema for one student be ₹ y.

So, $(x-8)(y+10)=xy$

$\Rightarrow\quad 5x-4y=40$ ...(i) $\frac{1}{2}$

and $(x+16)(y-10)=xy$

$\Rightarrow\quad 5x-8y=-80$ ...(ii) $\frac{1}{2}$

Here $A=\begin{bmatrix}5 & -4\\ 5 & -8\end{bmatrix}, X=\begin{bmatrix}x\\ y\end{bmatrix}, B=\begin{bmatrix}40\\ -80\end{bmatrix}$

$AX=B\Rightarrow X=A^{-1}B$

$A^{-1}=-\frac{1}{20}\begin{bmatrix}-8 & 4\\ -5 & 5\end{bmatrix}$ 1

$\Rightarrow\quad \begin{bmatrix}x\\ y\end{bmatrix}=\begin{bmatrix}32\\ 30\end{bmatrix}$

$\Rightarrow\quad x=32, y=30$ 1

No. of students = 32

Amount given to each student = ₹ 30.

Value reflected: To help needy people. 1

9. $\frac{dx}{dt}=e^{\cos 2t}(-2\sin 2t)$ or $-2x\sin 2t$ 1

$\frac{dy}{dt}=e^{\sin 2t}2\cos 2t$ or $2y\cos 2t$ 1

$$\frac{dy}{dx} = \frac{-e^{\sin 2t}\,2\cos 2t}{e^{\cos 2t}\,2\sin 2t} \text{ or } -\frac{y}{x}\frac{\cos 2t}{\sin 2t}$$ 1

$$= \frac{-y\log x}{x\log y}$$ 1

OR

$f(x) = 2\sin x + \sin 2x$ on $[0, \pi]$

$f(x)$ is continuous in $[0, \pi]$

$f(x)$ is differentiable in $(0, \pi)$ 1

$\therefore$ Mean value theorem is applicable

$f(0) = 0,\ f(\pi) = 0$

$f'(x) = 2\cos x + 2\cos 2x$ 1

$f'(c) = 2\cos c + 2\cos 2c$

$f'(c) = \dfrac{f(\pi) - f(0)}{\pi - 0} = 0$ 1

$\therefore$ $2\cos c + 2\cos 2c = 0$

$\Rightarrow$ $\cos c + 2\cos^2 c - 1 = 0$

$\Rightarrow$ $(2\cos c - 1)(\cos c + 1) = 0$

$\Rightarrow$ $\cos c = \dfrac{1}{2}$

$\Rightarrow$ $c = \dfrac{\pi}{3} \in (0, \pi)$

Hence mean value theorem is verified. $\frac{1}{2} + \frac{1}{2}$

10. $f(x) = \begin{cases} \dfrac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1} & x \neq 0 \\ -1 & x = 0 \end{cases}$

LHL: $\displaystyle\lim_{x \to 0^-} \frac{e^{\frac{1}{x}} - 1}{e^{\frac{1}{x}} + 1}$

$$= \lim_{h \to 0} \frac{e^{-\frac{1}{h}} - 1}{e^{-\frac{1}{h}} + 1} = \frac{0 - 1}{0 + 1} = -1$$ 2

RHL: $\displaystyle\lim_{h \to 0} \frac{e^{\frac{1}{h}} - 1}{e^{\frac{1}{h}} + 1} = \lim_{h \to 0} \frac{1 - e^{-\frac{1}{h}}}{1 + e^{-\frac{1}{h}}} = 1$ 2

LHL $\neq$ RHL

$\therefore$ $f(x)$ is discontinuous at $x = 0$

11. $y = \sqrt{5x-3} - 5$ ...(i)

$\dfrac{dy}{dx} = \dfrac{5}{2\sqrt{5x-3}}$ 1

Slope of line 4x – 2y + 5 = 0 is $\dfrac{-4}{-2} = 2.$ $\frac{1}{2}$

$\therefore \quad \dfrac{5}{2\sqrt{5x-3}} = 2 \qquad x \quad \dfrac{73}{80}$ 1

Putting $x = \dfrac{73}{80}$ in eqn. (i), we get $y = \dfrac{-15}{4}$ $\frac{1}{2}$

Equation of tangent

$y + \dfrac{15}{4} = 2\left(x - \dfrac{73}{80}\right)$ 1

or $\quad 80x - 40y - 223 = 0$

12. $\displaystyle\int_1^5 \{|x-1| + |x-2| + |x-3|\}\, dx$

$= \displaystyle\int_1^5 (x-1)\,dx + \int_1^2 (2-x)\,dx + \int_2^5 (x-2)\,dx + \int_1^3 (3-x)\,dx + \int_3^5 (x-3)\,dx$ $2\frac{1}{2}$

$= \left[\dfrac{x^2}{2} - x\right]_1^5 + \left[2x - \dfrac{x^2}{2}\right]_1^2 + \left[\dfrac{x^2}{2} - 2x\right]_2^5 + \left[3x - \dfrac{x^2}{2}\right]_1^3 + \left[\dfrac{x^2}{3} - 3x\right]_3^5$ 1

$= 17$ $\frac{1}{2}$

OR

Let $\quad I = \displaystyle\int_0^{\pi} \dfrac{x \sin x}{1 + 3\cos^2 x}\, dx$ ...(i)

$I = \displaystyle\int_0^{\pi} \dfrac{(\pi - x)\sin(\pi - x)}{1 + 3\cos^2(\pi - x)}\, dx$ 1

$= \displaystyle\int_0^{\pi} \dfrac{\pi \sin x}{1 + 3\cos^2 x}\, dx - \int_0^{\pi} \dfrac{x \sin x}{1 + 3\cos^2 x}\, dx$ ...(ii)

Adding (i) & (ii), we have

$2I = \displaystyle\int_0^{\pi} \dfrac{\pi \sin x}{1 + 3\cos^2 x}\, dx$ 1

Put cos x = t

–sin x dx = dt, when x = 0 ⇒ t = 1, for x = π ⇒ t = –1

$2I = -\pi \displaystyle\int_1^{-1} \dfrac{dt}{1 + 3t^2}$ 1

$$= \frac{\pi}{3}\int_{-1}^{1} \frac{dt}{\left(\frac{1}{\sqrt{3}}\right)^2 + (t)^2}$$

$$= \frac{\pi}{3}\times\sqrt{3}\left[\tan^{-1}(\sqrt{3}t\right]_{-1}^{1}$$

$$= \frac{\sqrt{3}\pi}{3}[\tan^{-1}\sqrt{3} - (-\tan^{-1}\sqrt{3})]$$

$$I = \frac{\sqrt{3}\pi}{3}\cdot\frac{\pi}{3} = \frac{\sqrt{3}\pi^2}{9}$$ 1

13. Let $I = \int \frac{2x+1}{(x^2+1)(x^2+4)}\,dx$

Let $\frac{2x+1}{(x^2+1)\,(x^2+4)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+4}$ 1

Getting $A = \frac{2}{3}, B = \frac{1}{3}, C = \frac{-2}{3}, D = \frac{-1}{3}$ 1

$\therefore$ $I = \frac{2}{3}\int\frac{x}{x^2+1}dx + \frac{1}{3}\int\frac{1}{x^2+1}dx + \frac{-2}{3}\int\frac{xdx}{x^2+4} + \frac{-1}{3}\int\frac{dx}{x^2+4}$

$$= \frac{1}{3}\log|x^2+1| + \frac{1}{3}\tan^{-1}x - \frac{1}{3}\log|x^2+4| - \frac{1}{6}\tan^{-1}\frac{x}{2} + C$$ 2

14. $(3x+5)\sqrt{5+4x-2x^2}\ dx$

let $3x + 5 = A(4 - 4x) + B$

$\Rightarrow$ $A = -\frac{3}{4}, B = 8$

$$I = -\frac{3}{4}\quad (4-4x)\sqrt{5+4x-2x^2}\ dx + 8\quad \sqrt{5+4x-2x^2}\ dx$$ 1

$$= -\frac{3}{4}I_1 + 8I_2 \text{ (let)}$$

For I_1, put $5 + 4x - 2x^2 = t$

$\Rightarrow$ $(4 - 4x)\,dx = dt$

$$-\frac{3}{4}I_1 = -\frac{3}{4}\quad \sqrt{t}\,dt = -\frac{3}{4}\times\frac{2}{3}t^{3/2}$$

$$= -\frac{1}{2}(5+4x-2x^2)^{3/2}$$ 1

$$8I_2 = 8\sqrt{2}\quad \sqrt{\frac{7}{2} - (x-1)^2}\ dx$$ $\frac{1}{2}$

$$= 8\sqrt{2}\left[\frac{x-1}{2}\sqrt{\frac{5}{2}+2x-x^2} + \frac{7}{4}\sin^{-1}\frac{\sqrt{2}(x-1)}{\sqrt{7}}\right]$$ 1

$$I = -\frac{1}{2}(5+4x-2x^2)^{3/2} + 4\sqrt{2}(x-1)\sqrt{\frac{5}{2}+2x-x^2} + 14\sqrt{2}\sin^{-1}\frac{\sqrt{2}(x-1)}{\sqrt{7}} + C$$ $\frac{1}{2}$

15. $x\dfrac{dy}{dx} + y - x + xy\cot x = 0$

$\Rightarrow \quad \dfrac{dy}{dx} + \left(\dfrac{1}{x} + \cot x\right)y = 1$ 1

$\text{I·F} = e^{\int\left(\frac{1}{x}+\cot x\right)dx} = e^{\log(x\sin x)}$ $\frac{1}{2}$

$= x\sin x$ 1

$\therefore \quad y \times x\sin x = \quad x\sin x\,dx$ $\frac{1}{2}$

$\Rightarrow \quad xy\sin x = -x\cos x + \sin x + C$ 1

16. $(x^2 + 3xy + y^2)\,dx - x^2\,dy = 0$

$\dfrac{dy}{dx} = \dfrac{x^2 + 3xy + y^2}{x^2}$ $\frac{1}{2}$

let $\quad y = vx$

$\dfrac{dy}{dx} = v + x\dfrac{dv}{dx}$ $\frac{1}{2}$

$\therefore \quad v + x\dfrac{dv}{dx} = 1 + 3v + v^2$

$\Rightarrow \quad x\dfrac{dv}{dx} = v^2 + 2v + 1$

$\Rightarrow \quad \dfrac{dv}{(v+1)^2} = \dfrac{dx}{x}$ 1

Integrating both sides

$\Rightarrow \quad -\dfrac{1}{v+1} = \log|x| + C$ 1

$\Rightarrow \quad \dfrac{-x}{x+y} = \log|x| + C$

When $x = 1, y = 0 \Rightarrow C = -1$ $\frac{1}{2}$

$\Rightarrow \quad \dfrac{-x}{x+y} = \log|x| - 1$

$\Rightarrow \quad y = (x + y)\log|x|$ $\frac{1}{2}$

or $\quad y = \dfrac{x\log|x|}{1 - \log|x|}$

17. $\vec{a} + \vec{b} = 5\hat{i} + \hat{k}$ $\frac{1}{2}$

$\vec{a} - \vec{b} = -\hat{i} - 2\hat{j} + 5\hat{k}$ $\frac{1}{2}$

Getting $\cos\theta = 0$ $1\frac{1}{2}$

$\Rightarrow \quad \theta = \dfrac{\pi}{2}$ $\frac{1}{2}$

a vector perpendicular to both $\vec{a} + \vec{b}$ & $\vec{a} - \vec{b}$ is $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = 2\hat{i} - 26\hat{j} - 10\hat{k}$ 1

18. $\frac{x-1}{3}=\frac{y-1}{-1}=\frac{z+1}{0}=\lambda$ (let) ...(i)

$\Rightarrow \quad x = 3\lambda + 1,\ y = -\lambda + 1,\ z = -1$ 1

$\frac{x-4}{2}=\frac{y}{0}=\frac{z+1}{3}=\mu$ ...(ii)

$\Rightarrow \quad x = 2\mu + 4,\ y = 0,\ z = 3\mu - 1$ 1

If the lines intersect, then they have a common point for some value of λ and μ.

So $\quad 3\lambda + 1 = 2\mu + 4$ ...(iii)

$-\lambda + 1 = 0 \Rightarrow \lambda = 1$

$3\mu - 1 = -1 \Rightarrow \mu = 0$ 1

Since $\lambda = 1$ & $\mu = 0$ satisfy equation (iii) so the given lines intersect and $\frac{1}{2}$

the point of intersection is (4, 0, –1). $\frac{1}{2}$

19. Let A = exactly 2 boys in the committee

B = at least one girl must be there in the committee.

$$P(B) = \frac{{}^4C_1 \times {}^7C_3 + {}^4C_2 \times {}^7C_2 + {}^4C_3 \times {}^7C_1 + {}^4C_4}{{}^{11}C_4}$$ 2

$$= \frac{59}{66}$$

$$P(A \cap B) = \frac{{}^4C_2 \times {}^7C_2}{{}^{11}C_4} = \frac{21}{55}$$ 1

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{21/55}{59/66} = \frac{126}{295}$$ 1

OR

$C + 2C + 2C + 3C + C^2 + 2C^2 + 7C^2 + C = 1$ 1

$\Rightarrow \quad 10C^2 + 9C = 1$

$\Rightarrow \quad 10C^2 + 9C - 1 = 0$

$\Rightarrow \quad C = \frac{1}{10}$ or $C = -1$ (not possible)

$\therefore \quad C = \frac{1}{10}$ 1

Mean $= 0 \times C + 1 \times 2C + 2 \times 2C + 3 \times 3C + 4 \times C^2 + 5 \times 2C^2 + 6(7C^2 + C)$ 1

$= 56C^2 + 21C$

$= 56 \times \frac{1}{100} + 21 \times \frac{1}{10}$

$= 0.56 + 2.1 = 2.66$ 1

SECTION C

20. (a, b) R(c, d) $\Rightarrow$ a + d = b + c

$\because$ a + b = b + a

$\Rightarrow$ (a, b) R(a, b) $\forall$ (a, b) $\in$ A $\times$ A

$\Rightarrow$ R is reflexive $\frac{1}{2}$

(a, b) R(c, d) $\Rightarrow$ a + d = b + c

$\Rightarrow$ b + c = a + d

$\Rightarrow$ c + b = d + a

$\Rightarrow$ (c, d) R(a, b)

$\Rightarrow$ R is symmetric $1\frac{1}{2}$

For (a, b), (c, d) & (e, f) $\in$ A $\times$ A

(a, b) R(c, d) $\Rightarrow$ a + d = b + c ...(1)

(c, d) R(e, f) $\Rightarrow$ c + f = d + e ...(2)

adding (1) & (2), we get

a + d + c + f = b + c + d + e

$\Rightarrow$ a + f = b + e

$\Rightarrow$ (a, b) R(e, f)

$\therefore$ R is transitive. $1\frac{1}{2}$

Hence R is an equivalence relation. $\frac{1}{2}$

Now [3, 4] = {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7), (7, 8), (8, 9), (9, 10)} 1

21. $$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a+x & a-x & a+x \end{vmatrix} = 0$$

Applying $C_1 \to C_1 + C_2 + C_3$ 1

$$\Rightarrow \begin{vmatrix} 3a-x & a-x & a-x \\ 3a-x & a+x & a-x \\ 3a-x & a-x & a+x \end{vmatrix} = 0$$ 1

$$\Rightarrow (3a-x)\begin{vmatrix} 1 & a-x & a-x \\ 1 & a+x & a-x \\ 1 & a-x & a+x \end{vmatrix} = 0$$ 1

Applying $R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$

$$\Rightarrow (3a-x)\begin{vmatrix} 1 & a-x & a-x \\ 0 & 2x & 0 \\ 0 & 0 & 2x \end{vmatrix} = 0$$ 2

$\Rightarrow$ $(3a-x)(4x^2) = 0$ 1

$\Rightarrow$ x = 0 or 3a

OR

$A = I \cdot A$ 1

$$\Rightarrow \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 - R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 4R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} A$$

$\left[2\frac{1}{2} \text{ for correct operations to get } A^{-1}\right]$

$$\therefore \quad A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$ $\frac{1}{2}$

The matrix form of given equations

$$\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 21 \\ 20 \\ 5 \end{bmatrix}$$ 1

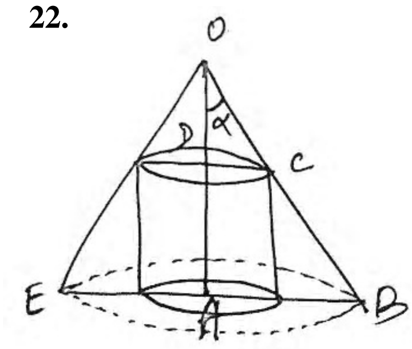
$$\Rightarrow \quad AX = B, \text{ where } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 21 \\ 20 \\ 5 \end{bmatrix}$$

$$\Rightarrow \quad X = A^{-1}B$$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} \begin{bmatrix} 21 \\ 20 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$$

$\therefore \quad x = 1, y = -2, z = 3$ 1

22.



Correct Figure 1

Let CD = R, AD = x

$\Rightarrow$ OD = h – x

$\because$ ODC ~ ΔOAB

$$\Rightarrow \frac{h-x}{h} = \frac{R}{AB} \Rightarrow \frac{h-x}{h} = \frac{R}{h\tan\alpha}$$

$\Rightarrow$ R = (h – x) tan α 1

$V = \pi R^2 x$ $\frac{1}{2}$

$= \pi(h-x)^2 \tan^2\alpha \cdot x$

$= \pi \tan^2\alpha (h-x)^2 x$ $\frac{1}{2}$

$$\frac{dV}{dx} = \pi \tan^2\alpha (h^2 - 4hx + 3x^2)$$ $\frac{1}{2}$

$$\frac{dV}{dx} = 0 \Rightarrow h^2 - 4hx + 3x^2 = 0$$

$$\Rightarrow (h-x)(h-3x) = 0$$

$\Rightarrow$ x = h (not possible) or $x = \frac{h}{3}$ 1

$$\frac{d^2V}{dx^2} = \pi \tan^2\alpha(-4h + 6x)$$

$$\left(\frac{d^2V}{dx^2}\right)_{x=h/3} = \pi\tan^2\alpha(-2h) < 0$$ 1

$\Rightarrow$ V is maximum for $x = \frac{h}{3}$

So $V_{max} = \pi \tan^2\alpha (h-x)^2 x$

$$= \pi \tan^2\alpha \left(h - \frac{h}{3}\right)^2 \frac{h}{3}$$

$$= \frac{4\pi h^3}{27} \tan^2\alpha$$ $\frac{1}{2}$

OR

$$y = \frac{4\sin x}{2+\cos x} - x, \quad x \in [0, 2\pi]$$

$$\frac{dy}{dx} = \frac{(2+\cos x)4\cos x - 4\sin x(-\sin x)}{(2+\cos x)^2} - 1$$ 2

$$\frac{dy}{dx} = \frac{\cos x(4-\cos x)}{(2+\cos x)^2}$$ 1

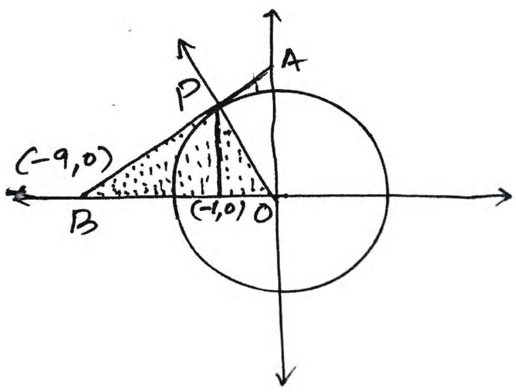
f(x) is strictly increasing for $f'(x) > 0$

i.e., $\cos x > 0 \Rightarrow x \in \left[0, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right]$ 2

and f(x) is strictly decreasing for $f'(x) < 0$

i.e., $\cos x < 0 \Rightarrow x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ 1

23.



Correct Figure 1

Equation of circle $x^2 + y^2 = 9$

Diff. w.r.t x, we have

$$\frac{dy}{dx} = -\frac{x}{y}$$ $\frac{1}{2}$

Slope of tangent at $\left(-1, 2\sqrt{2}\right)$

$$m_T = \left(-\frac{x}{y}\right)_{(-1, 2\sqrt{2})} = \frac{1}{2\sqrt{2}}$$ $\frac{1}{2}$

eqn. of tangent

$$y - 2\sqrt{2} = \frac{1}{2\sqrt{2}}(x+1)$$ 1

$$\Rightarrow \quad x - 2\sqrt{2}y + 9 = 0$$

It cuts x-axis at (–9, 0)

eqn. of normal

$$y - 2\sqrt{2} = -2\sqrt{2}(x+1)$$ 1

$$\Rightarrow \quad 2\sqrt{2}x + y = 0$$

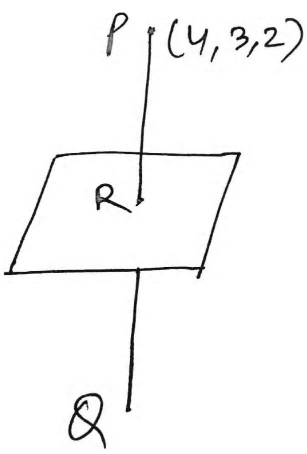
Area of ΔOPB

$$A = \int_{-9}^{-1} \frac{x+9}{2\sqrt{2}}\,dx + \int_{-1}^{0} -2\sqrt{2}x\,dx$$ 1

$$= \frac{1}{2\sqrt{2}}\left[\frac{x^2}{2} + 9x\right]_{-9}^{-1} - 2\sqrt{2}\left[\frac{x^2}{2}\right]_{-1}^{0}$$

$$= 9\sqrt{2} \text{ sq. unit}$$ 1

24.



Eqn. of plane $x + 2y + 3z = 2$

eqn. of PR is $\frac{x-4}{1} = \frac{y-3}{2} = \frac{z-3}{3} = \lambda$ (let) 1

$\Rightarrow x = \lambda + 4, y = 2\lambda + 3, z = 3\lambda + 2$ 1

Let the co-ordinate of R be $(\lambda + 4, 2\lambda + 3, 3\lambda + 2)$

R also lies on the plane

So, $\lambda + 4 + 2(2\lambda + 3) + 3(3\lambda + 2) = 2$

$\Rightarrow \lambda = -1$ 1

So point R is (3, 1, –1) i.e., foot of perpendicular 1

let Q(α, β, γ) be the image of P

$\therefore \quad \frac{4+\alpha}{2} = 3, \frac{3+\beta}{2} = 1, \frac{2+\gamma}{2} = -1$

$\Rightarrow \alpha = 2, \beta = -1, \gamma = -4$

So Image point Q is (2, –1, –4) 1

Perpendicular distance PR = $\sqrt{14}$ 1

25. $\left.\begin{array}{l} \text{P (winning)} = \frac{1}{9} \\ \text{P (not winning)} = \frac{8}{9} \end{array}\right\}$ 1

P(A winning) $= P(A) + P(\bar{A}\bar{B}\bar{C}A) + P(\bar{A}\bar{B}\bar{C}\bar{A}\bar{B}\bar{C}A) + ...$ 1

$= \frac{1}{9} + \left(\frac{8}{9}\right)^3 \frac{1}{9} + \left(\frac{8}{9}\right)^6 \frac{1}{9} + ...$

$= \frac{\frac{1}{9}}{1 - \frac{512}{729}} = \frac{81}{217}$ 1

P(B winning) $= P(\bar{A}B) + P(\bar{A}\bar{B}\bar{C}\bar{A}B) + P(\bar{A}\bar{B}\bar{C}\bar{A}\bar{B}\bar{C}\bar{A}B) + ...$ 1

$= \frac{8}{9} \times \frac{1}{9} + \left(\frac{8}{9}\right)^4 \times \frac{1}{9} + \left(\frac{8}{9}\right)^7 \times \frac{1}{9} + ...$

$= \frac{\frac{8}{9} \times \frac{1}{9}}{1 - \frac{512}{729}} = \frac{72}{217}$ 1

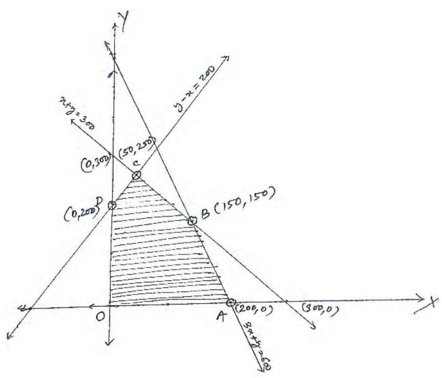
P(C winning) = 1 – [P(A winning) + P(B winning)]

$= 1 - \left[\frac{81}{217} + \frac{72}{217}\right]$

$= 1 - \frac{153}{217} = \frac{64}{217}$ 1

26. Let no. of cardigans of type A be x and that of type B by y.

Then, max. $Z = 100x + 50y$ 1



subject to constraint,

$x + y \leq 300$ ...(1)

$360x + 120y \leq 72{,}000$

$\Rightarrow$ $3x + y \leq 600$ ...(2)

$y - x \leq 200$ ...(3) 2

$x, y \geq 0$

Correct Figure 2

Corner points A(200, 0), B(150, 150), C(50, 250), D(0, 200), O(0, 0)

Corner points	**Z = 100x + 50y**
O(0, 0)	0
A(200, 0)	20,000
B(150, 150)	22,500 ← maximum
C(50, 250)	17,500
D(0, 200)	10,000

Hence no. of cardigans of type A = 150 and of type B = 150.

and max. profit is ₹ 22,500. 1

QUESTION PAPER CODE 65/2/E

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. 2^6 or 64 — 1

2. Finding $\cos\theta = \frac{\sqrt{3}}{2}$ — $\frac{1}{2}$

$|\vec{a} \times \vec{b}| = 6$ — $\frac{1}{2}$

3. $b_{21} = -16, b_{23} = -2$ [For any one correct value] — $\frac{1}{2}$

$b_{21} + b_{23} = -16 + (-2) = -18$ — $\frac{1}{2}$

4. $(\alpha, -\beta, \gamma)$ — 1

5. $R_1 \to R_1 + R_2 + R_3$ or $C_1 \to C_1 + C_2 + C_3$ — $\frac{1}{2}$

Ans. 0 — $\frac{1}{2}$

6. $\frac{1(\vec{a}-\vec{b})+3(\vec{a}+3\vec{b})}{4}$ (i.e., using correct formula) — $\frac{1}{2}$

$= \vec{a} + 2\vec{b}$ — $\frac{1}{2}$

SECTION B

7. Let the number of children be x and the amount distributed by Seema for one student be ₹ y.

So, $(x - 8)(y + 10) = xy$

$\Rightarrow \quad 5x - 4y = 40 \qquad \text{...(i)}$ — $\frac{1}{2}$

and $(x + 16)(y - 10) = xy$

$\Rightarrow \quad 5x - 8y = -80 \qquad \text{...(ii)}$ — $\frac{1}{2}$

Here $A = \begin{matrix} 5 & -4 \\ 5 & -8 \end{matrix}, X = \begin{matrix} x \\ y \end{matrix}, B = \begin{matrix} 40 \\ -80 \end{matrix}$

$AX = B \Rightarrow X = A^{-1}B$

$A^{-1} = -\frac{1}{20}\begin{matrix} -8 & 4 \\ -5 & 5 \end{matrix}$ — 1

$\Rightarrow \quad \begin{matrix} x \\ y \end{matrix} = \begin{matrix} 32 \\ 30 \end{matrix}$

$\Rightarrow \quad x = 32, y = 30$ — 1

No. of students = 32

Amount given to each student = ₹ 30.

Value reflected: To help needy people. 1

8. $f(x) = \begin{cases} \dfrac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1} & x \neq 0 \\ -1 & x = 0 \end{cases}$

LHL: $\lim\limits_{x\to 0^-} \dfrac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}$

$= \lim\limits_{h\to 0} \dfrac{e^{-\frac{1}{h}}-1}{e^{-\frac{1}{h}}+1} = \dfrac{0-1}{0+1} = -1$ 2

RHL: $\lim\limits_{h\to 0} \dfrac{e^{\frac{1}{h}}-1}{e^{\frac{1}{h}}+1} = \lim\limits_{h\to 0} \dfrac{1-e^{-\frac{1}{h}}}{1+e^{-\frac{1}{h}}} = 1$ 2

LHL $\neq$ RHL

$\therefore$ f(x) is discontinuous at x = 0

9. $\int_1^5 \{|x-1|+|x-2|+|x-3|\}\,dx$

$= \int_1^5 (x-1)\,dx + \int_1^2 (2-x)\,dx + \int_2^5 (x-2)\,dx + \int_1^3 (3-x)\,dx + \int_3^5 (x-3)\,dx$ $2\frac{1}{2}$

$= \left[\dfrac{x^2}{2}-x\right]_1^5 + \left[2x-\dfrac{x^2}{2}\right]_1^2 + \left[\dfrac{x^2}{2}-2x\right]_2^5 + \left[3x-\dfrac{x^2}{2}\right]_1^3 + \left[\dfrac{x^2}{3}-3x\right]_3^5$ 1

$= 17$ $\frac{1}{2}$

OR

Let $I = \int_0^\pi \dfrac{x\sin x}{1+3\cos^2 x}\,dx$ ...(i)

$I = \int_0^\pi \dfrac{(\pi-x)\sin(\pi-x)}{1+3\cos^2(\pi-x)}\,dx$ 1

$= \int_0^\pi \dfrac{\pi\sin x}{1+3\cos^2 x}\,dx - \int_0^\pi \dfrac{x\sin x}{1+3\cos^2 x}\,dx$ ...(ii)

Adding (i) & (ii), we have

$2I = \int_0^\pi \dfrac{\pi\sin x}{1+3\cos^2 x}\,dx$ 1

Put cos x = t

–sin x dx = dt, when x = 0 ⇒ t = 1, for x = π ⇒ t = –1

$$2I = -\pi\int_{1}^{-1}\frac{dt}{1+3t^2}$$ 1

$$= \frac{\pi}{3}\int_{-1}^{1}\frac{dt}{\left(\frac{1}{\sqrt{3}}\right)^2+(t)^2}$$

$$= \frac{\pi}{3}\times\sqrt{3}\left[\tan^{-1}(\sqrt{3}t\right]_{-1}^{1}$$

$$= \frac{\sqrt{3}\pi}{3}[\tan^{-1}\sqrt{3}-(-\tan^{-1}\sqrt{3})]$$

$$I = \frac{\sqrt{3}\pi}{3}\cdot\frac{\pi}{3} = \frac{\sqrt{3}\pi^2}{9}$$ 1

10. $(3x+5)\sqrt{5+4x-2x^2}\ dx$

let $3x + 5 = A(4 - 4x) + B$

⇒ $A = -\frac{3}{4}, B = 8$

$$I = -\frac{3}{4}\quad(4-4x)\sqrt{5+4x-2x^2}\ dx + 8\quad\sqrt{5+4x-2x^2}\ dx$$ 1

$$= -\frac{3}{4}I_1 + 8I_2 \text{ (let)}$$

For I_1, put $5 + 4x - 2x^2 = t$

⇒ $(4 - 4x)\ dx = dt$

$$-\frac{3}{4}I_1 = -\frac{3}{4}\quad\sqrt{t}\ dt = -\frac{3}{4}\times\frac{2}{3}t^{3/2}$$

$$= -\frac{1}{2}(5+4x-2x^2)^{3/2}$$ 1

$$8I_2 = 8\sqrt{2}\quad\sqrt{\frac{7}{2}-(x-1)^2}\ dx$$ $\frac{1}{2}$

$$= 8\sqrt{2}\left[\frac{x-1}{2}\sqrt{\frac{5}{2}+2x-x^2}+\frac{7}{4}\sin^{-1}\frac{\sqrt{2}(x-1)}{\sqrt{7}}\right]$$ 1

$$I = -\frac{1}{2}(5+4x-2x^2)^{3/2} + 4\sqrt{2}(x-1)\sqrt{\frac{5}{2}+2x-x^2} + 14\sqrt{2}\sin^{-1}\frac{\sqrt{2}(x-1)}{\sqrt{7}} + C$$ $\frac{1}{2}$

11. $(x^2 + 3xy + y^2)\ dx - x^2\ dy = 0$

$$\frac{dy}{dx} = \frac{x^2+3xy+y^2}{x^2}$$ $\frac{1}{2}$

let $y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$ $\frac{1}{2}$

$\therefore \quad v + x\frac{dv}{dx} = 1 + 3v + v^2$

$\Rightarrow \quad x\frac{dv}{dx} = v^2 + 2v + 1$

$\Rightarrow \quad \frac{dv}{(v+1)^2} = \frac{dx}{x}$ 1

Integrating both sides

$\Rightarrow \quad -\frac{1}{v+1} = \log|x| + C$ 1

$\Rightarrow \quad \frac{-x}{x+y} = \log|x| + C$

When $x = 1, y = 0 \Rightarrow C = -1$ $\frac{1}{2}$

$\Rightarrow \quad \frac{-x}{x+y} = \log|x| - 1$

$\Rightarrow \quad y = (x+y)\log|x|$ $\frac{1}{2}$

or $\quad y = \frac{x\log|x|}{1-\log|x|}$

12. $\tan^{-1}\left(\frac{2-x}{2+x}\right) = \frac{1}{2}\tan^{-1}\frac{x}{2}$

$\Rightarrow \quad 2\tan^{-1}\left(\frac{2-x}{2+x}\right) = \tan^{-1}\frac{x}{2}$ $\frac{1}{2}$

$\Rightarrow \quad \tan^{-1}\frac{2\left(\frac{2-x}{2+x}\right)}{1-\left(\frac{2-x}{2+x}\right)^2} = \tan^{-1}\frac{x}{2}$ $1\frac{1}{2}$

$\Rightarrow \quad \tan^{-1}\ \frac{4-x^2}{4x}\ = \tan^{-1}\frac{x}{2}$ 1

$\Rightarrow \quad \frac{4-x^2}{4x} = \frac{x}{2}$ $\frac{1}{2}$

$\Rightarrow \quad x = \frac{2}{\sqrt{3}} \quad (\because x > 0)$ $\frac{1}{2}$

OR

$2\sin^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{17}{31}\right)$

$= 2\tan^{-1}\left(\frac{3}{4}\right) - \tan^{-1}\left(\frac{17}{31}\right)$ 1

$= \tan^{-1}\frac{2\times\frac{3}{4}}{1-\left(\frac{3}{4}\right)^2} - \tan^{-1}\left(\frac{17}{31}\right)$ 1

$= \tan^{-1}\frac{24}{7} - \tan^{-1}\frac{17}{31}$

$$= \tan^{-1}\left(\frac{\frac{24}{7}-\frac{17}{31}}{1+\frac{24}{7}\times\frac{17}{31}}\right)$$ 1

$$= \tan^{-1}(1)$$ 1

$$= \frac{\pi}{4}$$

13. Let A = exactly 2 boys in the committee

B = at least one girl must be there in the committee.

$$P(B) = \frac{{}^4C_1 \times {}^7C_3 + {}^4C_2 \times {}^7C_2 + {}^4C_3 \times {}^7C_1 + {}^4C_4}{{}^{11}C_4}$$ 2

$$= \frac{59}{66}$$

$$P(A \cap B) = \frac{{}^4C_2 \times {}^7C_2}{{}^{11}C_4} = \frac{21}{55}$$ 1

$$P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{21/55}{59/66} = \frac{126}{295}$$ 1

OR

$C + 2C + 2C + 3C + C^2 + 2C^2 + 7C^2 + C = 1$ 1

$\Rightarrow \quad 10C^2 + 9C = 1$

$\Rightarrow \quad 10C^2 + 9C - 1 = 0$

$\Rightarrow \quad C = \frac{1}{10}$ or $C = -1$ (not possible)

$\therefore \quad C = \frac{1}{10}$ 1

Mean $= 0\times C + 1\times 2C + 2\times 2C + 3\times 3C + 4\times C^2 + 5\times 2C^2 + 6(7C^2 + C)$ 1

$= 56C^2 + 21C$

$= 56\times\frac{1}{100} + 21\times\frac{1}{10}$

$= 0.56 + 2.1 = 2.66$ 1

14. $\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0} = \lambda$ (let) ...(i)

$\Rightarrow \quad x = 3\lambda + 1,\ y = -\lambda + 1,\ z = -1$ 1

$\frac{x-4}{2} = \frac{y}{0} = \frac{z+1}{3} = \mu$ ...(ii)

$\Rightarrow \quad x = 2\mu + 4,\ y = 0,\ z = 3\mu - 1$ 1

If the lines intersect, then they have a common point for some value of λ and μ.

So $\quad 3\lambda + 1 = 2\mu + 4$ ...(iii)

$-\lambda + 1 = 0 \Rightarrow \lambda = 1$

$3\mu - 1 = -1 \Rightarrow \mu = 0$ 1

Since $\lambda = 1$ & $\mu = 0$ satisfy equation (iii) so the given lines intersect and $\frac{1}{2}$

the point of intersection is (4, 0, –1). $\frac{1}{2}$

15. $\frac{dx}{dt} = e^{\cos 2t}(-2\sin 2t)$ or $-2x \sin 2t$ 1

$\frac{dy}{dt} = e^{\sin 2t} 2\cos 2t$ or $2y \cos 2t$ 1

$\frac{dy}{dx} = \frac{-e^{\sin 2t} 2\cos 2t}{e^{\cos 2t} 2\sin 2t}$ or $-\frac{y}{x}\frac{\cos 2t}{\sin 2t}$ 1

$= \frac{-y\log x}{x\log y}$ 1

OR

$f(x) = 2\sin x + \sin 2x$ on $[0, \pi]$

$\left.\begin{array}{l} f(x) \text{ is continuous in } [0, \pi] \\ f(x) \text{ is differentiable in } (0, \pi) \end{array}\right\}$ 1

$\therefore$ Mean value theorem is applicable

$f(0) = 0, f(\pi) = 0$

$f'(x) = 2\cos x + 2\cos 2x$ 1

$f'(c) = 2\cos c + 2\cos 2c$

$f'(c) = \frac{f(\pi) - f(0)}{\pi - 0} = 0$ 1

$\therefore$ $2\cos c + 2\cos 2c = 0$

$\Rightarrow$ $\cos c + 2\cos^2 c - 1 = 0$

$\Rightarrow$ $(2\cos c - 1)(\cos c + 1) = 0$

$\Rightarrow$ $\cos c = \frac{1}{2}$

$\Rightarrow$ $c = \frac{\pi}{3} \in (0, \pi)$

Hence mean value theorem is verified. $\frac{1}{2} + \frac{1}{2}$

16. $\vec{a} + \vec{b} = 5\hat{i} + \hat{k}$ $\frac{1}{2}$

$\vec{a} - \vec{b} = -\hat{i} - 2\hat{j} + 5\hat{k}$ $\frac{1}{2}$

Getting $\cos\theta = 0$ $1\frac{1}{2}$

$\Rightarrow$ $\theta = \frac{\pi}{2}$ $\frac{1}{2}$

a vector perpendicular to both $\vec{a} + \vec{b}$ & $\vec{a} - \vec{b}$ is $(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = 2\hat{i} - 26\hat{j} - 10\hat{k}$ 1

17. $y = \sqrt{5x-3} - 5$ ...(i)

$\dfrac{dy}{dx} = \dfrac{5}{2\sqrt{5x-3}}$ 1

Slope of line 4x – 2y + 5 = 0 is $\dfrac{-4}{-2} = 2.$ $\frac{1}{2}$

$\therefore \quad \dfrac{5}{2\sqrt{5x-3}} = 2 \qquad x \quad \dfrac{73}{80}$ 1

Putting $x = \dfrac{73}{80}$ in eqn. (i), we get $y = \dfrac{-15}{4}$ $\frac{1}{2}$

Equation of tangent

$y + \dfrac{15}{4} = 2\left(x - \dfrac{73}{80}\right)$ 1

or $\quad 80x - 40y - 223 = 0$

18. $x\dfrac{dy}{dx} + y - x + xy\cot x = 0$

$\Rightarrow \quad \dfrac{dy}{dx} + \left(\dfrac{1}{x} + \cot x\right)y = 1$ 1

$I{\cdot}F = e^{\int\left(\frac{1}{x}+\cot x\right)dx} = e^{\log(x\sin x)}$ $\frac{1}{2}$

$= x\sin x$ 1

$\therefore \quad y \times x\sin x = \quad x\sin x\, dx$ $\frac{1}{2}$

$\Rightarrow \quad xy\sin x = -x\cos x + \sin x + C$ 1

19. Let $\quad I = \displaystyle\int \frac{2x+1}{(x^2+1)(x^2+4)}dx$

Let $\quad \dfrac{2x+1}{(x^2+1)\,(x^2+4)} = \dfrac{Ax+B}{x^2+1} + \dfrac{Cx+D}{x^2+4}$ 1

Getting $A = \dfrac{2}{3}, B = \dfrac{1}{3}, C = \dfrac{-2}{3}, D = \dfrac{-1}{3}$ 1

$\therefore \quad I = \dfrac{2}{3}\displaystyle\int\frac{x}{x^2+1}dx + \frac{1}{3}\int\frac{1}{x^2+1}dx + \frac{-2}{3}\int\frac{xdx}{x^2+4} + \frac{-1}{3}\int\frac{dx}{x^2+4}$

$= \dfrac{1}{3}\log|x^2+1| + \dfrac{1}{3}\tan^{-1}x - \dfrac{1}{3}\log|x^2+4| - \dfrac{1}{6}\tan^{-1}\dfrac{x}{2} + C$ 2

SECTION C

20. $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a+x & a-x & a+x \end{vmatrix} = 0$

Applying $C_1 \to C_1 + C_2 + C_3$ 1

$$\Rightarrow \quad \begin{vmatrix} 3a-x & a-x & a-x \\ 3a-x & a+x & a-x \\ 3a-x & a-x & a+x \end{vmatrix} = 0$$ 1

$$\Rightarrow \quad (3a-x)\begin{vmatrix} 1 & a-x & a-x \\ 1 & a+x & a-x \\ 1 & a-x & a+x \end{vmatrix} = 0$$ 1

Applying $R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$

$$\Rightarrow \quad (3a-x)\begin{vmatrix} 1 & a-x & a-x \\ 0 & 2x & 0 \\ 0 & 0 & 2x \end{vmatrix} = 0$$ 2

$\Rightarrow \quad (3a-x)(4x^2) = 0$ 1

$\Rightarrow \quad x = 0$ or $3a$

OR

$A = I \cdot A$ 1

$$\Rightarrow \quad \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 - R_2$

$$\Rightarrow \quad \begin{bmatrix} 1 & 0 & 0 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 4R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} A$$ $\left[2\frac{1}{2}\text{ for correct operations to get } A^{-1}\right]$

$$\therefore \quad A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$ $\frac{1}{2}$

The matrix form of given equations

$$\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 21 \\ 20 \\ 5 \end{bmatrix}$$ 1

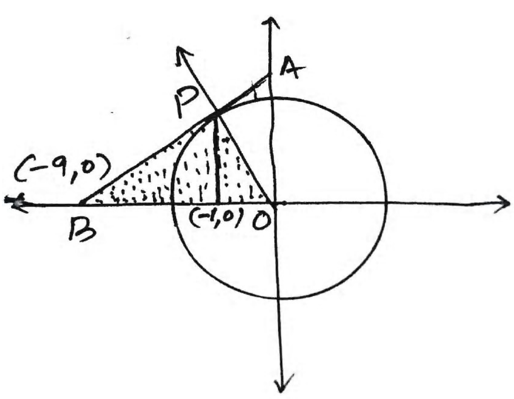
$\Rightarrow$ AX = B, where $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 21 \\ 20 \\ 5 \end{bmatrix}$

$\Rightarrow$ $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}\begin{bmatrix} 21 \\ 20 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$$

$\therefore$ $x = 1, y = -2, z = 3$ 1

21. Correct Figure 1



Equation of circle $x^2 + y^2 = 9$

Diff. w.r.t x, we have

$$\frac{dy}{dx} = -\frac{x}{y}$$ $\frac{1}{2}$

Slope of tangent at $\left(-1, 2\sqrt{2}\right)$

$$m_T = \left(-\frac{x}{y}\right)_{(-1,2\sqrt{2})} = \frac{1}{2\sqrt{2}}$$ $\frac{1}{2}$

eqn. of tangent

$$y - 2\sqrt{2} = \frac{1}{2\sqrt{2}}(x+1)$$ 1

$\Rightarrow$ $x - 2\sqrt{2}y + 9 = 0$

It cuts x-axis at (–9, 0)

eqn. of normal

$$y - 2\sqrt{2} = -2\sqrt{2}(x+1)$$ 1

$\Rightarrow$ $2\sqrt{2}x + y = 0$

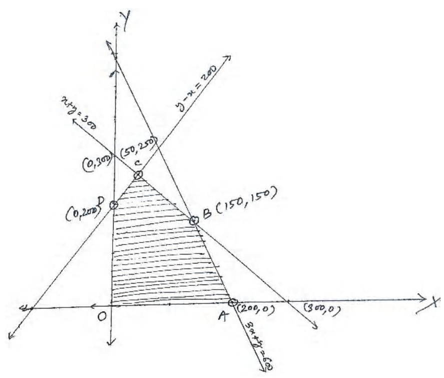
Area of ΔOPB

$$A = \int_{-9}^{-1} \frac{x+9}{2\sqrt{2}}\,dx + \int_{-1}^{0} -2\sqrt{2}x\,dx$$ 1

$$= \frac{1}{2\sqrt{2}}\left[\frac{x^2}{2} + 9x\right]_{-9}^{-1} - 2\sqrt{2}\left[\frac{x^2}{2}\right]_{-1}^{0}$$

$$= 9\sqrt{2} \text{ sq. unit}$$ 1

22. Let no. of cardigans of type A be x and that of type B by y.



Then, max. Z = 100x + 50y 1

subject to constraint,

$$x + y \le 300 \quad ...(1)$$

$$360x + 120y \le 72,000$$

$$\Rightarrow \quad 3x + y \le 600 \quad ...(2)$$

$$y - x \le 200 \quad ...(3) \qquad 2$$

$$x, y \ge 0$$

Correct Figure 2

Corner points A(200, 0), B(150, 150), C(50, 250), D(0, 200), O(0, 0)

Corner points	**Z = 100x + 50y**
O(0, 0)	0
A(200, 0)	20,000
B(150, 150)	22,500 ← maximum
C(50, 250)	17,500
D(0, 200)	10,000

Hence no. of cardigans of type A = 150 and of type B = 150.

and max. profit is ₹ 22,500. 1

23. $\left.\begin{array}{l} P\text{ (winning)} = \dfrac{1}{9} \\ \\ P\text{ (not winning)} = \dfrac{8}{9} \end{array}\right\}$ 1

$$P(\text{A winning}) = P(A) + P(\bar{A}\bar{B}\bar{C}A) + P(\bar{A}\bar{B}\bar{C}\bar{A}\bar{B}\bar{C}A) + ... \qquad 1$$

$$= \frac{1}{9} + \left(\frac{8}{9}\right)^3 \frac{1}{9} + \left(\frac{8}{9}\right)^6 \frac{1}{9} + ...$$

$$= \frac{\frac{1}{9}}{1 - \frac{512}{729}} = \frac{81}{217} \qquad 1$$

$$P(\text{B winning}) = P(\bar{A}B) + P(\bar{A}\bar{B}\bar{C}\bar{A}B) + P(\bar{A}\bar{B}\bar{C}\bar{A}\bar{B}\bar{C}\bar{A}B) + ... \qquad 1$$

$$= \frac{8}{9} \times \frac{1}{9} + \left(\frac{8}{9}\right)^4 \times \frac{1}{9} + \left(\frac{8}{9}\right)^7 \times \frac{1}{9} + ...$$

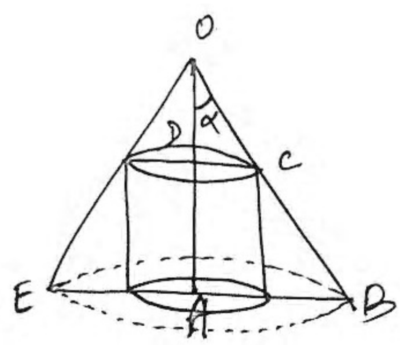
$$= \frac{\frac{8}{9} \times \frac{1}{9}}{1 - \frac{512}{729}} = \frac{72}{217} \qquad 1$$

$$P(\text{C winning}) = 1 - [P(\text{A winning}) + P(\text{B winning})]$$

$$= 1 - \left[\frac{81}{217} + \frac{72}{217}\right]$$

$$= 1 - \frac{153}{217} = \frac{64}{217}$$ 1

24. Correct Figure 1



Let CD = R, AD = x

$\Rightarrow$ OD = h – x

$\because$ ODC ~ ΔOAB

$$\Rightarrow \frac{h-x}{h} = \frac{R}{AB} \Rightarrow \frac{h-x}{h} = \frac{R}{h\tan\alpha}$$

$\Rightarrow$ R = (h – x) tan α 1

$V = \pi R^2 x$ $\frac{1}{2}$

$= \pi(h-x)^2 \tan^2\alpha \cdot x$

$= \pi \tan^2\alpha (h-x)^2 x$ $\frac{1}{2}$

$$\frac{dV}{dx} = \pi \tan^2\alpha (h^2 - 4hx + 3x^2)$$ $\frac{1}{2}$

$\frac{dV}{dx} = 0 \Rightarrow h^2 - 4hx + 3x^2 = 0$

$\Rightarrow (h-x)(h-3x) = 0$

$\Rightarrow$ x = h (not possible) or $x = \frac{h}{3}$ 1

$$\frac{d^2V}{dx^2} = \pi \tan^2\alpha(-4h + 6x)$$

$$\left(\frac{d^2V}{dx^2}\right)_{x=h/3} = \pi \tan^2\alpha(-2h) < 0$$ 1

$\Rightarrow$ V is maximum for $x = \frac{h}{3}$

So $v_{max} = \pi \tan^2\alpha (h-x)^2 x$

$$= \pi \tan^2\alpha \left(h - \frac{h}{3}\right)^2 \frac{h}{3}$$

$$= \frac{4\pi h^3}{27} \tan^2\alpha$$ $\frac{1}{2}$

OR

$$y = \frac{4\sin x}{2+\cos x} - x, \quad x \in [0, 2\pi]$$

$$\frac{dy}{dx} = \frac{(2+\cos x)4\cos x - 4\sin x(-\sin x)}{(2+\cos x)^2} - 1$$ 2

$$\frac{dy}{dx} = \frac{\cos x(4-\cos x)}{(2+\cos x)^2}$$ 1

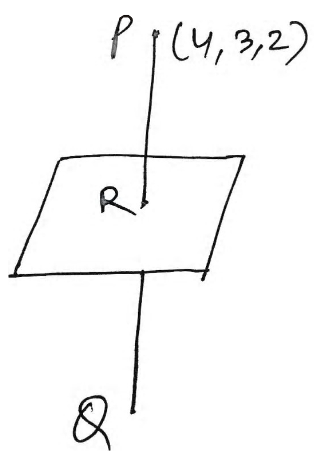
f(x) is strictly increasing for $f'(x) > 0$

i.e., $\cos x > 0 \Rightarrow x \in \left[0, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right]$ 2

and f(x) is strictly decreasing for $f'(x) < 0$

i.e., $\cos x < 0 \Rightarrow x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ 1

25.



Eqn. of plane x + 2y + 3z = 2

eqn. of PR is $\frac{x-4}{1} = \frac{y-3}{2} = \frac{z-3}{3} = \lambda$ (let) 1

$\Rightarrow x = \lambda + 4,\ y = 2\lambda + 3,\ z = 3\lambda + 2$ 1

Let the co-ordinate of R be $(\lambda + 4, 2\lambda + 3, 3\lambda + 2)$

R also lies on the plane

So, $\lambda + 4 + 2(2\lambda + 3) + 3(3\lambda + 2) = 2$

$\Rightarrow \lambda = -1$ 1

So point R is (3, 1, –1) i.e., foot of perpendicular 1

let $Q(\alpha, \beta, \gamma)$ be the image of P

$\therefore \frac{4+\alpha}{2} = 3, \frac{3+\beta}{2} = 1, \frac{2+\gamma}{2} = -1$

$\Rightarrow \alpha = 2, \beta = -1, \gamma = -4$

So Image point Q is (2, –1, –4) 1

Perpendicular distance PR = $\sqrt{14}$ 1

26. $(a, b)\,R(c, d) \Rightarrow a + d = b + c$

$\because \quad a + b = b + a$

$\Rightarrow \quad (a, b)\,R(a, b) \ \forall \ (a, b) \in A \times A$

$\Rightarrow \quad R$ is reflexive $\qquad \frac{1}{2}$

$(a, b)\,R(c, d) \quad \Rightarrow \ a + d = b + c$

$\Rightarrow \ b + c = a + d$

$\Rightarrow \ c + b = d + a$

$\Rightarrow \ (c, d)\,R(a, b)$

$\Rightarrow$ R is symmetric $\qquad 1\frac{1}{2}$

For $\ (a, b), (c, d)\ \&\ (e, f) \in A \times A$

$(a, b)\,R(c, d) \Rightarrow a + d = b + c \qquad ...(1)$

$(c, d)\,R(e, f) \Rightarrow c + f = d + e \qquad ...(2)$

adding (1) & (2), we get

$a + d + c + f = b + c + d + e$

$\Rightarrow \quad a + f = b + e$

$\Rightarrow \quad (a, b)\,R(e, f)$

$\therefore \quad$ R is transitive. $\qquad 1\frac{1}{2}$

Hence R is an equivalence relation. $\qquad \frac{1}{2}$

Now $[3, 4] = \{(1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7), (7, 8), (8, 9), (9, 10)\}$ $\qquad 1$

QUESTION PAPER CODE 65/3/E
EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $(\alpha, -\beta, \gamma)$ 1

2. $\dfrac{1(\vec{a}-\vec{b})+3(\vec{a}+3\vec{b})}{4}$ (i.e., using correct formula) $\frac{1}{2}$

$= \vec{a} + 2\vec{b}$ $\frac{1}{2}$

3. Finding $\cos\theta = \dfrac{\sqrt{3}}{2}$ $\frac{1}{2}$

$|\vec{a} \times \vec{b}| = 6$ $\frac{1}{2}$

4. $R_1 \to R_1 + R_2 + R_3$ or $C_1 \to C_1 + C_2 + C_3$ $\frac{1}{2}$

Ans. 0 $\frac{1}{2}$

5. $b_{21} = -16,\ b_{23} = -2$ [For any one correct value] $\frac{1}{2}$

$b_{21} + b_{23} = -16 + (-2) = -18$ $\frac{1}{2}$

6. 2^6 or 64 1

SECTION B

7. $y = \sqrt{5x-3} - 5$ ...(i)

$\dfrac{dy}{dx} = \dfrac{5}{2\sqrt{5x-3}}$ 1

Slope of line $4x - 2y + 5 = 0$ is $\dfrac{-4}{-2} = 2.$ $\frac{1}{2}$

$\therefore \quad \dfrac{5}{2\sqrt{5x-3}} = 2 \qquad x \quad \dfrac{73}{80}$ 1

Putting $x = \dfrac{73}{80}$ in eqn. (i), we get $y = \dfrac{-15}{4}$ $\frac{1}{2}$

Equation of tangent

$$y + \frac{15}{4} = 2\left(x - \frac{73}{80}\right)$$ 1

or $80x - 40y - 223 = 0$

8. $(x^2 + 3xy + y^2)\, dx - x^2\, dy = 0$

$$\frac{dy}{dx} = \frac{x^2 + 3xy + y^2}{x^2}$$ $\frac{1}{2}$

let $\quad y = vx$

$$\frac{dy}{dx} = v + x\frac{dv}{dx}$$ $\frac{1}{2}$

$\therefore \quad v + x\frac{dv}{dx} = 1 + 3v + v^2$

$\Rightarrow \quad x\frac{dv}{dx} = v^2 + 2v + 1$

$\Rightarrow \quad \frac{dv}{(v+1)^2} = \frac{dx}{x}$ 1

Integrating both sides

$\Rightarrow \quad -\frac{1}{v+1} = \log|x| + C$ 1

$\Rightarrow \quad \frac{-x}{x+y} = \log|x| + C$

When $x = 1, y = 0 \Rightarrow C = -1$ $\frac{1}{2}$

$\Rightarrow \quad \frac{-x}{x+y} = \log|x| - 1$

$\Rightarrow \quad y = (x + y)\log|x|$ $\frac{1}{2}$

or $\quad y = \frac{x\log|x|}{1 - \log|x|}$

9. Let the number of children be x and the amount distributed by Seema for one student be ₹ y.

So, $\quad (x - 8)(y + 10) = xy$

$\Rightarrow \quad 5x - 4y = 40 \qquad$ ...(i) $\frac{1}{2}$

and $\quad (x + 16)(y - 10) = xy$

$\Rightarrow \quad 5x - 8y = -80 \qquad$ ...(ii) $\frac{1}{2}$

Here $A = \begin{bmatrix} 5 & -4 \\ 5 & -8 \end{bmatrix}, X = \begin{bmatrix} x \\ y \end{bmatrix}, B = \begin{bmatrix} 40 \\ -80 \end{bmatrix}$

$AX = B \Rightarrow X = A^{-1}B$

$A^{-1} = -\frac{1}{20}\begin{bmatrix} -8 & 4 \\ -5 & 5 \end{bmatrix}$ 1

$\Rightarrow \quad \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 32 \\ 30 \end{bmatrix}$

$\Rightarrow \quad x = 32, y = 30$ 1

No. of students = 32

Amount given to each student = ₹ 30.

Value reflected: To help needy people. 1

10. $\frac{x-1}{3} = \frac{y-1}{-1} = \frac{z+1}{0} = \lambda$ (let) ...(i)

$\Rightarrow$ $x = 3\lambda + 1, y = -\lambda + 1, z = -1$ 1

$\frac{x-4}{2} = \frac{y}{0} = \frac{z+1}{3} = \mu$ ...(ii)

$\Rightarrow$ $x = 2\mu + 4, y = 0, z = 3\mu - 1$ 1

If the lines intersect, then they have a common point for some value of λ and μ.

So $3\lambda + 1 = 2\mu + 4$ ...(iii)

$-\lambda + 1 = 0 \Rightarrow \lambda = 1$

$3\mu - 1 = -1 \Rightarrow \mu = 0$ 1

Since $\lambda = 1$ & $\mu = 0$ satisfy equation (iii) so the given lines intersect and $\frac{1}{2}$

the point of intersection is (4, 0, –1). $\frac{1}{2}$

11. $f(x) = \begin{cases} \dfrac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1} & x \neq 0 \\ -1 & x = 0 \end{cases}$

LHL: $\lim\limits_{x\to 0^-} \dfrac{e^{\frac{1}{x}}-1}{e^{\frac{1}{x}}+1}$

$= \lim\limits_{h\to 0} \dfrac{e^{-\frac{1}{h}}-1}{e^{-\frac{1}{h}}+1} = \dfrac{0-1}{0+1} = -1$ 2

RHL: $\lim\limits_{h\to 0} \dfrac{e^{\frac{1}{h}}-1}{e^{\frac{1}{h}}+1} = \lim\limits_{h\to 0} \dfrac{1-e^{-\frac{1}{h}}}{1+e^{-\frac{1}{h}}} = 1$ 2

LHL $\neq$ RHL

$\therefore$ f(x) is discontinuous at x = 0

12. Let $I = \int \frac{2x+1}{(x^2+1)(x^2+4)} dx$

Let $\frac{2x+1}{(x^2+1)(x^2+4)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+4}$ 1

Getting $A = \frac{2}{3}, B = \frac{1}{3}, C = \frac{-2}{3}, D = \frac{-1}{3}$ 1

$\therefore$ $I = \frac{2}{3}\int \frac{x}{x^2+1}dx + \frac{1}{3}\int \frac{1}{x^2+1}dx + \frac{-2}{3}\int \frac{xdx}{x^2+4} + \frac{-1}{3}\int \frac{dx}{x^2+4}$

$= \frac{1}{3}\log|x^2+1| + \frac{1}{3}\tan^{-1}x - \frac{1}{3}\log|x^2+4| - \frac{1}{6}\tan^{-1}\frac{x}{2} + C$ 2

13. $\dfrac{dx}{dt} = e^{\cos 2t}(-2\sin 2t)$ or $-2x \sin 2t$ 1

$\dfrac{dy}{dt} = e^{\sin 2t}\, 2\cos 2t$ or $2y \cos 2t$ 1

$\dfrac{dy}{dx} = \dfrac{-e^{\sin 2t}\, 2\cos 2t}{e^{\cos 2t}\, 2\sin 2t}$ or $-\dfrac{y}{x}\dfrac{\cos 2t}{\sin 2t}$ 1

$= \dfrac{-y\log x}{x\log y}$ 1

OR

$f(x) = 2 \sin x + \sin 2x$ on $[0, \pi]$

$\left.\begin{array}{l} f(x) \text{ is continuous in } [0, \pi] \\ f(x) \text{ is differentiable in } (0, \pi) \end{array}\right\}$ 1

$\therefore$ Mean value theorem is applicable

$f(0) = 0,\ f(\pi) = 0$

$f'(x) = 2 \cos x + 2 \cos 2x$ 1

$f'(c) = 2 \cos c + 2 \cos 2c$

$f'(c) = \dfrac{f(\pi) - f(0)}{\pi - 0} = 0$ 1

$\therefore$ $2 \cos c + 2 \cos 2c = 0$

$\Rightarrow$ $\cos c + 2 \cos 2\, c - 1 = 0$

$\Rightarrow$ $(2 \cos c - 1)(\cos c + 1) = 0$

$\Rightarrow$ $\cos c = \dfrac{1}{2}$

$\Rightarrow$ $c = \dfrac{\pi}{3} \in (0, \pi)$

Hence mean value theorem is verified. $\frac{1}{2} + \frac{1}{2}$

14. $\tan^{-1}\left(\dfrac{2-x}{2+x}\right) = \dfrac{1}{2}\tan^{-1}\dfrac{x}{2}$

$\Rightarrow$ $2\tan^{-1}\left(\dfrac{2-x}{2+x}\right) = \tan^{-1}\dfrac{x}{2}$ $\frac{1}{2}$

$\Rightarrow$ $\tan^{-1}\dfrac{2\left(\dfrac{2-x}{2+x}\right)}{1-\left(\dfrac{2-x}{2+x}\right)^2} = \tan^{-1}\dfrac{x}{2}$ $1\frac{1}{2}$

$\Rightarrow$ $\tan^{-1}\dfrac{4-x^2}{4x} = \tan^{-1}\dfrac{x}{2}$ 1

$\Rightarrow$ $\dfrac{4-x^2}{4x} = \dfrac{x}{2}$ $\frac{1}{2}$

$\Rightarrow$ $x = \dfrac{2}{\sqrt{3}}$ $(\because x > 0)$ $\frac{1}{2}$

OR

$$2\sin^{-1}\left(\frac{3}{5}\right)-\tan^{-1}\left(\frac{17}{31}\right)$$

$$=2\tan^{-1}\left(\frac{3}{4}\right)-\tan^{-1}\left(\frac{17}{31}\right)$$ 1

$$=\tan^{-1}\frac{2\times\frac{3}{4}}{1-\left(\frac{3}{4}\right)^2}-\tan^{-1}\left(\frac{17}{31}\right)$$ 1

$$=\tan^{-1}\frac{24}{7}-\tan^{-1}\frac{17}{31}$$

$$=\tan^{-1}\left(\frac{\frac{24}{7}-\frac{17}{31}}{1+\frac{24}{7}\times\frac{17}{31}}\right)$$ 1

$$=\tan^{-1}(1)$$ 1

$$=\frac{\pi}{4}$$

15. $$\int_1^5\{|x-1|+|x-2|+|x-3|\}\,dx$$

$$=\int_1^5(x-1)\,dx+\int_1^2(2-x)\,dx+\int_2^5(x-2)\,dx+\int_1^3(3-x)\,dx+\int_3^5(x-3)\,dx$$ $2\frac{1}{2}$

$$=\left[\frac{x^2}{2}-x\right]_1^5+\left[2x-\frac{x^2}{2}\right]_1^2+\left[\frac{x^2}{2}-2x\right]_2^5+\left[3x-\frac{x^2}{2}\right]_1^3+\left[\frac{x^2}{3}-3x\right]_3^5$$ 1

$$=17$$ $\frac{1}{2}$

OR

Let $$I=\int_0^{\pi}\frac{x\sin x}{1+3\cos^2x}\,dx \quad ...(i)$$

$$I=\int_0^{\pi}\frac{(\pi-x)\sin(\pi-x)}{1+3\cos^2(\pi-x)}\,dx$$ 1

$$=\int_0^{\pi}\frac{\pi\sin x}{1+3\cos^2x}\,dx-\int_0^{\pi}\frac{x\sin x}{1+3\cos^2x}\,dx$$ ...(ii)

Adding (i) & (ii), we have

$$2I=\int_0^{\pi}\frac{\pi\sin x}{1+3\cos^2x}\,dx$$ 1

Put $\cos x = t$

$-\sin x\,dx = dt$, when $x = 0 \Rightarrow t = 1$, for $x = \pi \Rightarrow t = -1$

$$2I = -\pi\int_{1}^{-1}\frac{dt}{1+3t^2}$$ 1

$$= \frac{\pi}{3}\int_{-1}^{1}\frac{dt}{\left(\frac{1}{\sqrt{3}}\right)^2 + (t)^2}$$

$$= \frac{\pi}{3}\times\sqrt{3}\left[\tan^{-1}(\sqrt{3}t\right]_{-1}^{1}$$

$$= \frac{\sqrt{3}\pi}{3}[\tan^{-1}\sqrt{3} - (-\tan^{-1}\sqrt{3})]$$

$$I = \frac{\sqrt{3}\pi}{3}\cdot\frac{\pi}{3} = \frac{\sqrt{3}\pi^2}{9}$$ 1

16. $x\frac{dy}{dx} + y - x + xy\cot x = 0$

$\Rightarrow \quad \frac{dy}{dx} + \left(\frac{1}{x} + \cot x\right)y = 1$ 1

$I{\cdot}F = e^{\int\left(\frac{1}{x}+\cot x\right)dx} = e^{\log(x\sin x)}$ $\frac{1}{2}$

$= x\sin x$ 1

$\therefore \quad y \times x\sin x = \quad x\sin x\ dx$ $\frac{1}{2}$

$\Rightarrow \quad xy\sin x = -x\cos x + \sin x + C$ 1

17. Let A = exactly 2 boys in the committee

B = at least one girl must be there in the committee.

$$P(B) = \frac{{}^4C_1\times{}^7C_3 + {}^4C_2\times{}^7C_2 + {}^4C_3\times{}^7C_1 + {}^4C_4}{{}^{11}C_4}$$ 2

$$= \frac{59}{66}$$

$$P(A\cap B) = \frac{{}^4C_2\times{}^7C_2}{{}^{11}C_4} = \frac{21}{55}$$ 1

$$P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{21/55}{59/66} = \frac{126}{295}$$ 1

OR

$C + 2C + 2C + 3C + C^2 + 2C^2 + 7C^2 + C = 1$ 1

$\Rightarrow \quad 10C^2 + 9C = 1$

$\Rightarrow \quad 10C^2 + 9C - 1 = 0$

$\Rightarrow \quad C = \frac{1}{10}$ or $C = -1$ (not possible)

$\therefore \quad C = \frac{1}{10}$ 1

Mean $= 0\times C+1\times 2C+2\times 2C+3\times 3C+4\times C^2+5\times 2C^2+6(7C^2+C)$ 1

$= 56C^2 + 21C$

$= 56\times\frac{1}{100}+21\times\frac{1}{10}$

$= 0.56+2.1=2.66$ 1

18. $\vec{a}+\vec{b}=5\hat{i}+\hat{k}$ $\frac{1}{2}$

$\vec{a}-\vec{b}=-\hat{i}-2\hat{j}+5\hat{k}$ $\frac{1}{2}$

Getting $\cos\theta = 0$ $1\frac{1}{2}$

$\Rightarrow \quad \theta=\frac{\pi}{2}$ $\frac{1}{2}$

a vector perpendicular to both $\vec{a}+\vec{b}$ & $\vec{a}-\vec{b}$ is $(\vec{a}+\vec{b})\times(\vec{a}-\vec{b})=2\hat{i}-26\hat{j}-10\hat{k}$ 1

19. $(3x+5)\sqrt{5+4x-2x^2}\ dx$

let $3x + 5 = A(4 - 4x) + B$

$\Rightarrow \quad A=-\frac{3}{4}, B=8$

$I=-\frac{3}{4}\quad(4-4x)\sqrt{5+4x-2x^2}\ dx+8\quad\sqrt{5+4x-2x^2}\ dx$ 1

$=-\frac{3}{4}I_1+8I_2$ (let)

For I_1, put $5 + 4x - 2x^2 = t$

$\Rightarrow \quad (4 - 4x)\ dx = dt$

$-\frac{3}{4}I_1=-\frac{3}{4}\quad\sqrt{t}\ dt=-\frac{3}{4}\times\frac{2}{3}t^{3/2}$

$=-\frac{1}{2}(5+4x-2x^2)^{3/2}$ 1

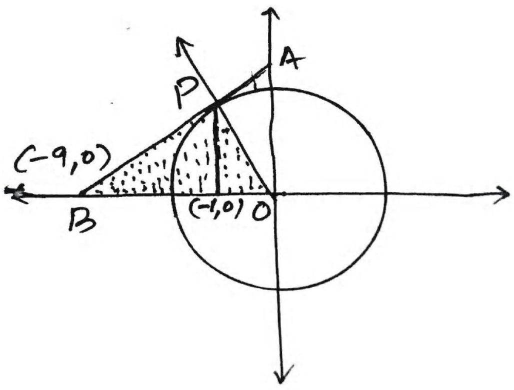
$8I_2 = 8\sqrt{2}\quad\sqrt{\frac{7}{2}-(x-1)^2}\ dx$ $\frac{1}{2}$

$=8\sqrt{2}\left[\frac{x-1}{2}\sqrt{\frac{5}{2}+2x-x^2}+\frac{7}{4}\sin^{-1}\frac{\sqrt{2}(x-1)}{\sqrt{7}}\right]$ 1

$I=-\frac{1}{2}(5+4x-2x^2)^{3/2}+4\sqrt{2}(x-1)\sqrt{\frac{5}{2}+2x-x^2}+14\sqrt{2}\sin^{-1}\frac{\sqrt{2}(x-1)}{\sqrt{7}}+C$ $\frac{1}{2}$

SECTION C

20. Correct Figure 1



Equation of circle $x^2 + y^2 = 9$

Diff. w.r.t x, we have

$$\frac{dy}{dx} = -\frac{x}{y}$$ $\frac{1}{2}$

Slope of tangent at $\left(-1, 2\sqrt{2}\right)$

$$m_T = \left(-\frac{x}{y}\right)_{(-1, 2\sqrt{2})} = \frac{1}{2\sqrt{2}}$$ $\frac{1}{2}$

eqn. of tangent

$$y - 2\sqrt{2} = \frac{1}{2\sqrt{2}}(x+1)$$ 1

$$\Rightarrow \quad x - 2\sqrt{2}y + 9 = 0$$

It cuts x-axis at (–9, 0)

eqn. of normal

$$y - 2\sqrt{2} = -2\sqrt{2}(x+1)$$ 1

$$\Rightarrow \quad 2\sqrt{2}x + y = 0$$

Area of ΔOPB

$$A = \int_{-9}^{-1} \frac{x+9}{2\sqrt{2}}\,dx + \int_{-1}^{0} -2\sqrt{2}x\,dx$$ 1

$$= \frac{1}{2\sqrt{2}}\left[\frac{x^2}{2} + 9x\right]_{-9}^{-1} - 2\sqrt{2}\left[\frac{x^2}{2}\right]_{-1}^{0}$$

$$= 9\sqrt{2} \text{ sq. unit}$$ 1

21. Let no. of cardigans of type A be x and that of type B by y.
Then, max. $Z = 100x + 50y$ 1

subject to constraint,

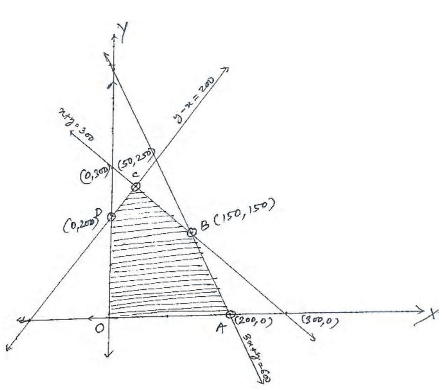
$x + y \le 300$ ...(1)

$360x + 120y \le 72{,}000$

$\Rightarrow$ $3x + y \le 600$ ...(2)

$y - x \le 200$ ...(3) 2

$x, y \ge 0$



Correct Figure 2

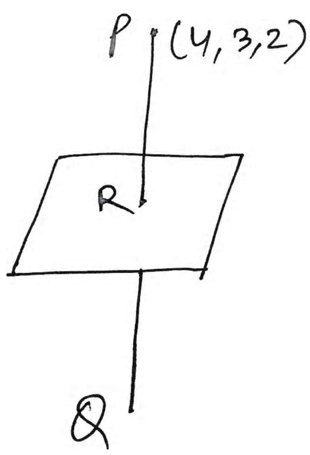
Corner points A(200, 0), B(150, 150), C(50, 250), D(0, 200), O(0, 0)

Corner points	**Z = 100x + 50y**
O(0, 0)	0
A(200, 0)	20,000
B(150, 150)	22,500 ← maximum
C(50, 250)	17,500
D(0, 200)	10,000

Hence no. of cardigans of type A = 150 and of type B = 150.

and max. profit is ₹ 22,500. 1

22.



Eqn. of plane x + 2y + 3z = 2

eqn. of PR is $\frac{x-4}{1}=\frac{y-3}{2}=\frac{z-3}{3}=\lambda$ (let) 1

$\Rightarrow\ x = \lambda + 4,\ y = 2\lambda + 3,\ z = 3\lambda + 2$ 1

Let the co-ordinate of R be $(\lambda + 4, 2\lambda + 3, 3\lambda + 2)$

R also lies on the plane

So, $\lambda + 4 + 2(2\lambda + 3) + 3(3\lambda + 2) = 2$

$\Rightarrow\ \lambda = -1$ 1

So point R is (3, 1, –1) i.e., foot of perpendicular 1

let $Q(\alpha, \beta, \gamma)$ be the image of P

$\therefore\ \frac{4+\alpha}{2}=3,\ \frac{3+\beta}{2}=1,\ \frac{2+\gamma}{2}=-1$

$\Rightarrow\ \alpha = 2,\ \beta = -1,\ \gamma = -4$

So Image point Q is (2, –1, –4) 1

Perpendicular distance PR = $\sqrt{14}$ 1

23. $\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a+x & a-x & a+x \end{vmatrix}=0$

Applying $C_1 \to C_1 + C_2 + C_3$ 1

$$\Rightarrow\ \begin{vmatrix} 3a-x & a-x & a-x \\ 3a-x & a+x & a-x \\ 3a-x & a-x & a+x \end{vmatrix}=0$$ 1

$$\Rightarrow\ (3a-x)\begin{vmatrix} 1 & a-x & a-x \\ 1 & a+x & a-x \\ 1 & a-x & a+x \end{vmatrix}=0$$ 1

Applying $R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$

$$\Rightarrow\ (3a-x)\begin{vmatrix} 1 & a-x & a-x \\ 0 & 2x & 0 \\ 0 & 0 & 2x \end{vmatrix}=0$$ 2

$\Rightarrow \quad (3a - x)(4x^2) = 0$ 1

$\Rightarrow \quad x = 0 \text{ or } 3a$

OR

$A = I \cdot A$ 1

$$\Rightarrow \begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_1 \to R_1 - R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 2R_1$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_2 \to R_2 - 4R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ 0 & 0 & 1 \end{bmatrix} A$$

Applying $R_3 \to R_3 + R_2$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} A$$

$\left[2\frac{1}{2} \text{ for correct operations to get } A^{-1}\right]$

$$\therefore \quad A^{-1} = \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix}$$ $\frac{1}{2}$

The matrix form of given equations

$$\begin{bmatrix} 3 & -3 & 4 \\ 2 & -3 & 4 \\ 0 & -1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 21 \\ 20 \\ 5 \end{bmatrix}$$ 1

$$\Rightarrow \quad AX = B, \text{ where } X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 21 \\ 20 \\ 5 \end{bmatrix}$$

$\Rightarrow \quad X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} 1 & -1 & 0 \\ -2 & 3 & -4 \\ -2 & 3 & -3 \end{bmatrix} \begin{bmatrix} 21 \\ 20 \\ 5 \end{bmatrix} = \begin{bmatrix} 1 \\ -2 \\ 3 \end{bmatrix}$$

$\therefore \quad x = 1, y = -2, z = 3$ 1

24. $\left.\begin{array}{l} P\text{ (winning)} = \frac{1}{9} \\ \\ P\text{ (not winning)} = \frac{8}{9} \end{array}\right\}$ 1

$P(\text{A winning}) = P(A) + P(\bar{A}\bar{B}\bar{C}A) + P(\bar{A}\bar{B}\bar{C}\bar{A}\bar{B}\bar{C}A) + ...$ 1

$$= \frac{1}{9} + \left(\frac{8}{9}\right)^3 \frac{1}{9} + \left(\frac{8}{9}\right)^6 \frac{1}{9} + ...$$

$$= \frac{\frac{1}{9}}{1 - \frac{512}{729}} = \frac{81}{217}$$ 1

$P(\text{B winning}) = P(\bar{A}B) + P(\bar{A}\bar{B}\bar{C}\bar{A}B) + P(\bar{A}\bar{B}\bar{C}\bar{A}\bar{B}\bar{C}\bar{A}B) + ...$ 1

$$= \frac{8}{9} \times \frac{1}{9} + \left(\frac{8}{9}\right)^4 \times \frac{1}{9} + \left(\frac{8}{9}\right)^7 \times \frac{1}{9} + ...$$

$$= \frac{\frac{8}{9} \times \frac{1}{9}}{1 - \frac{512}{729}} = \frac{72}{217}$$ 1

$P(\text{C winning}) = 1 - [P(\text{A winning}) + P(\text{B winning})]$

$$= 1 - \left[\frac{81}{217} + \frac{72}{217}\right]$$

$$= 1 - \frac{153}{217} = \frac{64}{217}$$ 1

25. $(a, b)\ R(c, d) \Rightarrow a + d = b + c$

$\because$ $a + b = b + a$

$\Rightarrow$ $(a, b)\ R(a, b)\ \forall\ (a, b) \in A \times A$

$\Rightarrow$ R is reflexive $\frac{1}{2}$

$(a, b)\ R(c, d) \Rightarrow a + d = b + c$

$\Rightarrow b + c = a + d$

$\Rightarrow c + b = d + a$

$\Rightarrow (c, d)\ R(a, b)$

$\Rightarrow$ R is symmetric $1\frac{1}{2}$

For $(a, b), (c, d) \ \&\ (e, f) \in A \times A$

$(a, b)\ R(c, d) \Rightarrow a + d = b + c$ ...(1)

$(c, d)\ R(e, f) \Rightarrow c + f = d + e$ ...(2)

adding (1) & (2), we get

$a + d + c + f = b + c + d + e$

$\Rightarrow \quad a + f = b + e$

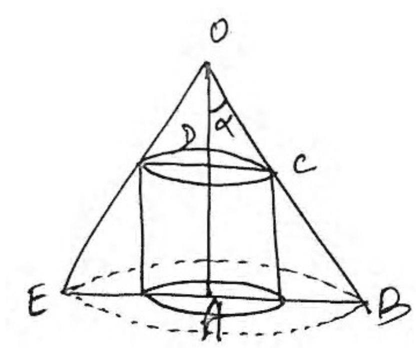
$\Rightarrow \quad (a, b)\ R(e, f)$

$\therefore \quad$ R is transitive. $\quad 1\frac{1}{2}$

Hence R is an equivalence relation. $\quad \frac{1}{2}$

Now [3, 4] = {(1, 2), (2, 3), (3, 4), (4, 5), (5, 6), (6, 7), (7, 8), (8, 9), (9, 10)} $\quad 1$

26. Correct Figure $\quad 1$



Let $CD = R,\ AD = x$

$\Rightarrow \quad OD = h - x$

$\because \quad ODC \sim \Delta OAB$

$$\Rightarrow \quad \frac{h-x}{h} = \frac{R}{AB} \Rightarrow \frac{h-x}{h} = \frac{R}{h\tan\alpha}$$

$\Rightarrow \quad R = (h - x)\tan\alpha \quad 1$

$V = \pi R^2 x \quad \frac{1}{2}$

$= \pi(h - x)^2 \tan^2 \alpha \cdot x$

$= \pi \tan^2 \alpha (h - x)^2 x \quad \frac{1}{2}$

$$\frac{dV}{dx} = \pi\tan^2\alpha(h^2 - 4hx + 3x^2) \quad \frac{1}{2}$$

$$\frac{dV}{dx} = 0 \quad \Rightarrow \quad h^2 - 4hx + 3x^2 = 0$$

$\Rightarrow \quad (h - x)(h - 3x) = 0$

$\Rightarrow \quad x = h$ (not possible) or $x = \frac{h}{3} \quad 1$

$$\frac{d^2V}{dx^2} = \pi\tan^2\alpha(-4h + 6x)$$

$$\left(\frac{d^2V}{dx^2}\right)_{x=h/3} = \pi\tan^2\alpha(-2h) < 0 \quad 1$$

$\Rightarrow \quad$ V is maximum for $x = \frac{h}{3}$

So $\quad V_{max} = \pi \tan^2 \alpha (h - x)^2 x$

$$= \pi\tan^2\alpha\left(h - \frac{h}{3}\right)^2 \frac{h}{3}$$

$$= \frac{4\pi h^3}{27}\tan^2\alpha \quad \frac{1}{2}$$

OR

$$y = \frac{4\sin x}{2+\cos x} - x, \quad x \in [0, 2\pi]$$

$$\frac{dy}{dx} = \frac{(2+\cos x)4\cos x - 4\sin x(-\sin x)}{(2+\cos x)^2} - 1$$ 2

$$\frac{dy}{dx} = \frac{\cos x(4-\cos x)}{(2+\cos x)^2}$$ 1

f(x) is strictly increasing for $f'(x) > 0$

i.e., $\cos x > 0 \Rightarrow x \in \left[0, \frac{\pi}{2}\right) \cup \left(\frac{3\pi}{2}, 2\pi\right]$ 2

and f(x) is strictly decreasing for $f'(x) < 0$

i.e., $\cos x < 0 \Rightarrow x \in \left(\frac{\pi}{2}, \frac{3\pi}{2}\right)$ 1