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Senior School Certificate Examination

March 2016

Marking Scheme — Mathematics 65/1/S, 65/2/S, 65/3/S

General Instructions:

1. The Marking Scheme provides general guidelines to reduce subjectivity in the marking. The answers given in the Marking Scheme are suggested answers. The content is thus indicative. If a student has given any other answer which is different from the one given in the Marking Scheme, but conveys the meaning, such answers should be given full weightage

2. Evaluation is to be done as per instructions provided in the marking scheme. It should not be done according to one's own interpretation or any other consideration — Marking Scheme should be strictly adhered to and religiously followed.

3. Alternative methods are accepted. Proportional marks are to be awarded.

4. In question (s) on differential equations, constant of integration has to be written.

5. If a candidate has attempted an extra question, marks obtained in the question attempted first should be retained and the other answer should be scored out.

6. A full scale of marks - 0 to 100 has to be used. Please do not hesitate to award full marks if the answer deserves it.

7. Separate Marking Scheme for all the three sets has been given.

8. As per orders of the Hon'ble Supreme Court. The candidates would now be permitted to obtain photocopy of the Answer book on request on payment of the prescribed fee. All examiners/Head Examiners are once again reminded that they must ensure that evaluation is carried out strictly as per value points for each answer as given in the Marking Scheme.

QUESTION PAPER CODE 65/1/S
EXPECTED ANSWER/VALUE POINTS

SECTION A

1. Getting $\sin\theta = \dfrac{\frac{1}{\sqrt{3}}}{\frac{1}{2}\cdot\frac{4}{\sqrt{3}}} = \dfrac{1}{2}$ $\qquad \frac{1}{2}$

Hence $|\vec{a}.\vec{b}| = \dfrac{1}{2}\cdot\dfrac{4}{\sqrt{3}}\cdot\dfrac{\sqrt{3}}{2} = 1$ $\qquad \frac{1}{2}$

2. $|\vec{a} - \sqrt{2}\vec{b}|^2 = 1 \Rightarrow \vec{a}.\vec{b} = \dfrac{1}{\sqrt{2}}.$ $\qquad \frac{1}{2}$

$\therefore$ Angle between $\vec{a}$ and $\vec{b}$ = $\dfrac{\pi}{4}$ $\qquad \frac{1}{2}$

3. Writing or using, that given planes are parallel $\qquad \frac{1}{2}$

$d = \dfrac{|4+10|}{\sqrt{4+9+36}} = 2$ units $\qquad \frac{1}{2}$

4. $|AA^T| = |A||A^T| = |A|^2$ $\qquad \frac{1}{2}$

$= 25$ $\qquad \frac{1}{2}$

5. Getting AB = $\begin{pmatrix} 7 & -8 \\ 0 & -10 \end{pmatrix}$ $\qquad \frac{1}{2}$

$|AB| = -70$ $\qquad \frac{1}{2}$

6. $k(2) = -8 \Rightarrow k = -4$ $\qquad \frac{1}{2}$

$-4(3) = 4a \Rightarrow a = -3$ $\qquad \frac{1}{2}$

SECTION B

7. $y = (\sin 2x)^x + \sin^{-1}(\sqrt{3x}) = u + v$

$\therefore \quad \dfrac{dy}{dx} = \dfrac{du}{dx} + \dfrac{dv}{dx}$ $\qquad \frac{1}{2}$

$u = (\sin 2x)^x \Rightarrow \log u = x \log \sin 2x$ $\qquad \frac{1}{2}$

$\dfrac{1}{u}\dfrac{du}{dx} = 2x \cdot \cot 2x + \log \sin 2x$ $\qquad 1$

$\therefore \quad \frac{du}{dx} = (\sin 2x)^x [2x\cot 2x + \log \sin 2x]$ $\frac{1}{2}$

$\frac{dv}{dx} = \frac{1}{\sqrt{1-3x}} \frac{\sqrt{3}}{2\sqrt{x}}$ 1

$\therefore \quad \frac{dy}{dx} = (\sin 2x)^x [2x\cot 2x + \log \sin 2x] + \frac{\sqrt{3}}{2\sqrt{x}\sqrt{1-3x}}$ $\frac{1}{2}$

OR

Let $y = \tan^{-1}\left(\frac{\sqrt{1+x^2}-\sqrt{1-x^2}}{\sqrt{1+x^2}+\sqrt{1-x^2}}\right)$ and $z = \cos^{-1} x^2$

$z = \cos^{-1} x^2 \Rightarrow x^2 = \cos z \Rightarrow y = \tan^{-1}\left(\frac{\sqrt{1+\cos z}-\sqrt{1-\cos z}}{\sqrt{1+\cos z}+\sqrt{1-\cos z}}\right)$ 1

$\therefore \quad y = \tan^{-1}\left(\frac{\cos\frac{z}{2}-\sin\frac{z}{2}}{\cos\frac{z}{2}+\sin\frac{z}{2}}\right) = \tan^{-1}\left(\frac{1-\tan\frac{z}{2}}{1+\tan\frac{z}{2}}\right)$ $\frac{1}{2} + \frac{1}{2}$

$\therefore \quad y = \tan^{-1}\left[\tan\left(\frac{\pi}{4}-\frac{z}{2}\right)\right] = \frac{\pi}{4}-\frac{z}{2}$ $\frac{1}{2} + \frac{1}{2}$

$\Rightarrow \quad \frac{dy}{dz} = -\frac{1}{2}$ 1

8. LHL $= \lim_{x\to 0^-} k.\sin\frac{\pi}{2}(x+1) = k$ 1

RHL $= \lim_{x\to 0^+} \frac{\tan x(1-\cos x)}{x^3}$ 1

$= \lim_{x\to 0^+} \frac{\tan x}{x}.2\left(\frac{\sin x/2}{2x/2}\right)^2 = \frac{1}{2}$ 1

$\Rightarrow \quad k = \frac{1}{2}$ 1

9. When $x = am^2$, we get $y = \pm am^3$ 1

$ay^2 = x^3 \Rightarrow 2ay\frac{dy}{dx} = 3x^2 \Rightarrow \frac{dy}{dx} = \frac{3x^2}{2ay}$ 1

slope of normal $= \mp\frac{2a}{3}\frac{am^3}{a^2m^4} = \mp\frac{2}{3m}$ 1

$\therefore$ Equation of normal is $y \mp am^3 = \mp\frac{2}{3m}(x-am^2)$ 1

[Full marks may be given, if only one value for point, slope and equation is derived]

10. Writing $\int \frac{1-\sin x}{\sin x(1+\sin x)}\,dx = \int \frac{(1+\sin x)-2\sin x}{\sin x(1+\sin x)}\,dx$ 1

$= \int \frac{1}{\sin x}\,dx - 2\int \frac{1}{1+\sin x}\,dx$ 1

$= \int \operatorname{cosec} x\,dx - 2\int \frac{(1-\sin x)}{\cos^2 x}\,dx$ 1

$= \log|\operatorname{cosec} x - \cot x| - 2\int (\sec^2 x - \sec x \tan x)\,dx$ $\frac{1}{2}$

$= \log|\operatorname{cosec} x - \cot x| - 2(\tan x - \sec x) + C$ $\frac{1}{2}$

11. $I = \int \left[\log(\log x) + \frac{1}{(\log x)^2}\right] dx$

$= \int \log(\log x).1\,dx + \int \frac{1}{(\log x)^2}\,dx$ 1

$= \log(\log x)\cdot x - \int \frac{1}{\log x}\cdot\frac{1}{\cancel{x}}\cdot\cancel{x}\,dx + \int \frac{1}{(\log x)^2}\,dx$ 2

$= x\cdot\log(\log x) - \left[\frac{1}{\log x}\cdot x - \int \frac{-1}{(\log x)^2}\cdot\frac{1}{x}\cdot x\,dx\right] + \int \frac{1}{(\log x)^2}\,dx$ $\frac{1}{2}$

$= x\log(\log x) - \frac{x}{\log x} + C$ $\frac{1}{2}$

12. $I = \int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x}\,dx$ ...(i)

$I = \int_0^{\pi/2} \frac{\sin^2(\pi/2 - x)}{\sin(\pi/2 - x) + \cos(\pi/2 - x)}\,dx = \int_0^{\pi/2} \frac{\cos^2 x}{\cos x + \sin x}\,dx$ ...(ii) 1

$2I = \int_0^{\pi/2} \frac{1}{\sin x + \cos x}\,dx$ 1

$\Rightarrow I = \frac{1}{2\sqrt{2}} \int_0^{\pi/2} \frac{1}{\cos x \frac{1}{\sqrt{2}} + \sin x \frac{1}{\sqrt{2}}}\,dx = \frac{1}{2\sqrt{2}} \int_0^{\pi/2} \sec\left(x - \frac{\pi}{4}\right) dx$ 1

$= \frac{1}{2\sqrt{2}}\left[\log\left|\sec\left(x-\frac{\pi}{4}\right) + \tan\left(x - \frac{\pi}{4}\right)\right|\right]_0^{\pi/2}$ $\frac{1}{2}$

$= \frac{1}{2\sqrt{2}} \log\left|\frac{\sqrt{2}+1}{\sqrt{2}-1}\right|$ or $\frac{1}{\sqrt{2}} \log|\sqrt{2}+1|$ $\frac{1}{2}$

OR

$$\text{I} = \int_0^1 \cot^{-1}(1-x+x^2)\,dx = \int_0^1 \tan^{-1}\left(\frac{1}{1-x+x^2}\right)dx$$ $\frac{1}{2}$

$$= \int_0^1 \tan^{-1}\left(\frac{x+(1-x)}{1-x(1-x)}\right)dx = \int_0^1 \tan^{-1} x\,dx + \int_0^1 \tan^{-1}(1-x)\,dx$$ 1

$$= 2\int_0^1 \tan^{-1} x\,dx$$ $\frac{1}{2}$

$$= 2\left[\left(\tan^{-1} x . x\right)_0^1 - \int_0^1 \frac{x}{1+x^2}\,dx\right]$$ $\frac{1}{2}$

$$= 2\left[x\tan^{-1} x - \frac{1}{2}\log|1+x^2|\right]_0^1$$ 1

$$= 2\left[\frac{\pi}{4} - \frac{1}{2}\log 2\right] \text{ or } \frac{\pi}{2} - \log 2$$ $\frac{1}{2}$

13. The given differential equation can be written as

$$\frac{dy}{dx} - \frac{1}{x+1}\,y = (x+1)^2.e^{3x}$$ $\frac{1}{2}$

Here, integrating factor = $e^{\int -\frac{1}{x+1}dx} = \frac{1}{x+1}$ 1

$\therefore$ Solution is $y\frac{1}{x+1} = \int (x+1)\,e^{3x} dx$ 1

$$\therefore \quad \frac{y}{x+1} = (x+1)\frac{e^{3x}}{3} - \frac{e^{3x}}{9} + \text{C}$$ $1\frac{1}{2}$

or $$y = \left[\frac{1}{3}(x+1)^2 - \frac{x+1}{9}\right]e^{3x} + \text{C}(x+1)$$

14. From the given differential equation, we can write

$$\frac{dx}{dy} = \frac{2xe^{x/y} - y}{2ye^{x/y}} = \frac{2\,x/y\;e^{x/y} - 1}{2e^{x/y}}$$ 1

Putting $\frac{x}{y} = v \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$ $\frac{1}{2}$

$$\therefore \quad v + y\frac{dv}{dy} = \frac{2ve^v - 1}{2e^v} \Rightarrow y\frac{dv}{dy} = -\frac{1}{2e^v}$$ 1

$$\Rightarrow \quad 2\int e^v\,dv = -\int \frac{dy}{y}$$ $\frac{1}{2}$

$$\therefore \quad 2e^v + \log|y| = \text{C} \Rightarrow 2e^{x/y} + \log|y| = \text{C}$$ 1

15. Let length be x m and breadth be y m

$\therefore \quad (x-50)(y+50) = xy \Rightarrow 50x - 50y = 2500$ or $x - y = 50$ $\quad \frac{1}{2}$

and $\quad (x-10)(y-20) = xy - 5300 \Rightarrow 2x + y = 550$ $\quad \frac{1}{2}$

$$\begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 50 \\ 550 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 & 1 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 50 \\ 550 \end{pmatrix} \qquad \frac{1}{2} + 1$$

$$\Rightarrow \quad x = \frac{1}{3}(600) = 200 \text{ m}, \; y = \frac{1}{3}(450) = 150 \text{ m} \qquad \frac{1}{2}$$

"Helping the children of his village to learn" (or any other relavent value) 1

16. LHS $= 2\sin^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{17}{31}\right)$

$$= 2\tan^{-1}\left(\frac{3}{4}\right) - \tan^{-1}\left(\frac{17}{31}\right) \qquad 1$$

$$= \tan^{-1}\left(\frac{2\cdot\frac{3}{4}}{1-\frac{9}{16}}\right) - \tan^{-1}\left(\frac{17}{31}\right) = \tan^{-1}\left(\frac{24}{7}\right) - \tan^{-1}\left(\frac{17}{31}\right) \qquad 1$$

$$\tan^{-1}\left(\frac{\frac{24}{7}-\frac{17}{31}}{1+\frac{24}{7}\cdot\frac{17}{31}}\right) = \tan^{-1}\left(\frac{625}{625}\right) = \tan^{-1}(1) = \frac{\pi}{4} = \text{RHS} \qquad 1 + 1$$

OR

$$\cos(\tan^{-1} x) = \sin\left(\cot^{-1}\frac{3}{4}\right)$$

$$\Rightarrow \quad \cos\left(\cos^{-1}\frac{1}{\sqrt{1+x^2}}\right) = \sin\left(\sin^{-1}\frac{4}{5}\right) \qquad 1 + 1$$

$$\Rightarrow \quad \frac{1}{\sqrt{1+x^2}} = \frac{4}{5} \text{ or } \sqrt{1+x^2} = \frac{5}{4} \qquad 1$$

$$\Rightarrow \quad x^2 = \frac{9}{16} \Rightarrow x = \pm\frac{3}{4} \qquad 1$$

17. Let E_1 : selecting bag A, E_2 : selecting bag B $\quad \frac{1}{2}$

A : getting 2 white and 1 red out of 3 drawn (without replacement)

$\therefore \; P(E_1) = P(E_2) = \frac{1}{2}$ $\quad \frac{1}{2}$

$$P\left(\frac{A}{E_1}\right) = \frac{{}^3C_2 \cdot {}^4C_1}{{}^7C_3} = \frac{12}{35}$$

1

$$P\left(\frac{A}{E_2}\right) = \frac{{}^4C_2 \cdot {}^3C_1}{{}^7C_3} = \frac{18}{35}$$ 1

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)\,P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{2}\cdot\frac{18}{35}}{\frac{1}{2}\cdot\frac{12}{35} + \frac{1}{2}\cdot\frac{18}{35}} = \frac{3}{5}$$ 1

18. $\vec{a} = \vec{b} + \vec{c} \Rightarrow p\hat{i} + q\hat{j} + r\hat{k} = (s+3)\hat{i} + 4\hat{j} + 2\hat{k}$

$p = s + 3,\ q = 4,\ r = 2$ $1\frac{1}{2}$

area $= \frac{1}{2}|\vec{b} \times \vec{c}| = 5\sqrt{6}$

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ s & 3 & 4 \\ 3 & 1 & -2 \end{vmatrix} = -10\hat{i} + (2s+12)\hat{j} + (s-9)\hat{k}$$ $\frac{1}{2}$

$\therefore\ 100 + (2s + 12)^2 + (s - 9)^2 = (10\sqrt{6})^2 = 600$

$\Rightarrow s^2 + 6s + 55 = 0 \Rightarrow s = -11,\ p = -8,$ or $s = 5,\ p = 8$ 1 + 1

19. Equation of plane passing through A, B and C is

$$\begin{vmatrix} x-3 & y-2 & z-1 \\ 1 & 0 & -3 \\ 2 & 3 & 1 \end{vmatrix} = 0$$ 2

$\Rightarrow (x - 3)9 - (y - 2)7 + (z - 1)3 = 0 \Rightarrow 9x - 7y + 3z = 16$ ...(i) 1

If A, B, C and D are coplanar, D must lie on (i)

$\Rightarrow 9\lambda - 35 + 15 - 16 = 0 \Rightarrow \lambda = 4.$ 1

OR

Equation of plane, perpendicular to $\vec{n} = \hat{i} + \hat{j} + 3\hat{k}$ and at a distance $\frac{4}{\sqrt{11}}$ from origin is

$\vec{r} \cdot \frac{(\hat{i} + \hat{j} + 3\hat{k})}{\sqrt{11}} = \frac{4}{\sqrt{11}}$ or $\vec{r} \cdot (\hat{i} + \hat{j} + 3\hat{k}) = 4$ ...(i) $1\frac{1}{2}$

Any point on the line $\vec{r} = (-\hat{i} - 2\hat{j} - 3\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 3\hat{k})$ is

$(-1+3\lambda)\hat{i} + (-2+4\lambda)\hat{j} + (-3+3\lambda)\hat{k}$ ...(ii) 1

If this point is the point of intersection of the plane and the line then,

$(-1+3\lambda)1 + (-2+4\lambda)1 + (-3+3\lambda)3 = 4$

$\Rightarrow \lambda = 1.$ 1

Hence the point of intersection is (2, 2, 0) $\frac{1}{2}$

SECTION C

20. Let $x_1, x_2 \in \mathrm{N}$ and $f(x_1) = f(x_2)$

$\Rightarrow 4x_1^2 + 12x_1 + 15 = 4x_2^2 + 12x_2 + 15$

$\Rightarrow 4(x_1^2 - x_2^2) + 12(x_1 - x_2) = 0 \Rightarrow (x_1 - x_2)(4x_1 + 4x_2 + 12) = 0$

$\Rightarrow x_1 - x_2 = 0$ or $x_1 = x_2$ as $4x_1 + 4x_2 + 12 \neq 0$, $x_1, x_2 \in \mathrm{N}$

$\therefore f$ is a $1-1$ function

2

$f : \mathrm{N} \to \mathrm{S}$ is onto as co-domain = range 1

Hence f is invertible.

$$y = 4x^2 + 12x + 15 = (2x+3)^2 + 6 \Rightarrow x = \frac{\sqrt{y-6}-3}{2}$$

$$\therefore \quad f^{-1}(y) = \frac{\sqrt{y-6}-3}{2}, \; y \in \mathrm{S}.$$ 2

$$f^{-1}(31) = \frac{\sqrt{31-6}-3}{2} = 1$$ $\frac{1}{2}$

$$f^{-1}(87) = \frac{\sqrt{87-6}-3}{2} = 3$$ $\frac{1}{2}$

21. Let $\Delta = \begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix}$

$C_1 \to C_1 + C_2 - 2C_3 \Rightarrow \Delta = (a^2+b^2+c^2)\begin{vmatrix} 1 & a^2 & bc \\ 1 & b^2 & ca \\ 1 & c^2 & ab \end{vmatrix}$ 1

$R_1 \to R_1 - R_2$, and $R_2 \to R_2 - R_3 \Rightarrow \Delta = (a^2+b^2+c^2)\begin{vmatrix} 0 & a^2-b^2 & c(b-a) \\ 0 & b^2-c^2 & a(c-b) \\ 1 & c^2 & ab \end{vmatrix}$ $1\frac{1}{2}$

$= (a^2+b^2+c^2)(a-b)(b-c)\begin{vmatrix} 0 & a+b & -c \\ 0 & b+c & -a \\ 1 & c^2 & ab \end{vmatrix}$ $\frac{1}{2}$

$R_2 \to R_2 - R_1 \Rightarrow \Delta = (a^2+b^2+c^2)(a-b)(b-c)\begin{vmatrix} 0 & a+b & -c \\ 0 & c-a & c-a \\ 1 & c^2 & ab \end{vmatrix}$ 1

$\therefore \Delta = (a^2+b^2+c^2)(a-b)(b-c)(c-a)\begin{vmatrix} 0 & a+b & -c \\ 0 & 1 & 1 \\ 1 & c^2 & ab \end{vmatrix}$ 1

Expanding by C_1 to get $\Delta = (a^2+b^2+c^2)(a-b)(b-c)(c-a)(a+b+c)$ 1

OR

Let A = IA $\therefore$ $\begin{pmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$ 1

$R_2 \to R_2 - 2R_3 \Rightarrow \begin{pmatrix} 2 & -1 & 3 \\ 1 & -1 & -5 \\ -3 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix} A$ $\frac{1}{2}$

$\begin{matrix} R_1 \to R_1 - 2R_2 \\ R_3 \to R_3 + 3R_2 \end{matrix} \Rightarrow \begin{pmatrix} 0 & 1 & 13 \\ 1 & -1 & -5 \\ 0 & -1 & -12 \end{pmatrix} = \begin{pmatrix} 1 & -2 & 4 \\ 0 & 1 & -2 \\ 0 & 3 & -5 \end{pmatrix} A$ 1

$R_1 \leftrightarrow R_2 \Rightarrow \begin{pmatrix} 1 & -1 & -5 \\ 0 & 1 & 13 \\ 0 & -1 & -12 \end{pmatrix} = \begin{pmatrix} 0 & 1 & -2 \\ 1 & -2 & 4 \\ 0 & 3 & -5 \end{pmatrix} A$ $\frac{1}{2}$

$\begin{cases} R_1 \to R_1 + R_2 \\ R_3 \to R_3 + R_2 \end{cases} \Rightarrow \begin{pmatrix} 1 & 0 & 8 \\ 0 & 1 & 13 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 2 \\ 1 & -2 & 4 \\ 1 & 1 & -1 \end{pmatrix} A$ 1

$\begin{cases} R_1 \to R_1 - 8R_3 \\ R_2 \to R_2 - 13R_3 \end{cases} \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -7 & -9 & 10 \\ -12 & -15 & 17 \\ 1 & 1 & -1 \end{pmatrix} A$ 1

$\Rightarrow A^{-1} = \begin{pmatrix} -7 & -9 & 10 \\ -12 & -15 & 17 \\ 1 & 1 & -1 \end{pmatrix}$ 1

22. $f'(x) = 4x^3 - 24x^2 + 44x - 24$ 1

$= 4(x^3 - 6x^2 + 11x - 6) = 4(x - 1)(x - 2)(x - 3)$ $1\frac{1}{2}$

$f'(x) = 0 \Rightarrow x = 1, x = 2, x = 3$ $\frac{1}{2}$

The intervals are $(-\infty, 1), (1, 2), (2, 3), (3, \infty)$ 1

since $f'(x) > 0$ in $(1, 2)$ and $(3, \infty)$

$\therefore$ $f(x)$ is strictly increasing in $(1, 2) \cup (3, \infty)$ 1

and strictly decreasing in $(-\infty, 1) \cup (2, 3)$ 1

OR

$f(x) = \sec x + 2 \log |\cos x|$

$f'(x) = \sec x \tan x - 2 \tan x = \tan x (\sec x - 2)$ 1

$f'(x) = 0 \Rightarrow \tan x = 0$ or $\sec x = 2$

$\Rightarrow x = \pi, x = \frac{\pi}{3}, \frac{5\pi}{3}$ $1\frac{1}{2}$

$f''(x) = \sec x \tan^2 x + (\sec x - 2) \sec^2 x$ 1

$f''(\pi/3) = 6$ (+ve) $\Rightarrow f(x)$ is minimum at $x = \pi/3$

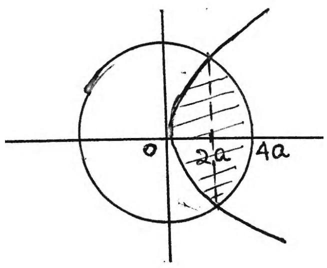
$f''(\pi) = -3$ (–ve) $\Rightarrow f(x)$ is maximum at $x = \pi$ $\quad 1\frac{1}{2}$

$f''(5\pi/3) = 6$ (+ve) $\Rightarrow f(x)$ is minimum at $x = 5\pi/3$

Maximum value $= f(\pi) = -1$. $\quad \frac{1}{2}$

Minimum value $= f(\pi/3) = f(5\pi/3) = 2 - 2 \log 2$ or $2 + \log (1/4)$ $\quad \frac{1}{2}$

23.



Solving $y^2 = 6ax$ and $x^2 + y^2 = 16a^2$

we get $x^2 + 6ax - 16a^2 = 0$

$(x + 8a)(x - 2a) = 0$

$x = -8a,\ x = 2a$ $\quad 1$

Correct Figure $\quad 1$

Required area $= 2\left[\int_0^{2a} \sqrt{6a}\sqrt{x}\, dx + \int_{2a}^{4a} \sqrt{16a^2 - x^2}\, dx\right]$ $\quad 2$

$$= 2\left[\left(\sqrt{6}\sqrt{a}\,\frac{2}{3}x^{3/2}\right)_0^{2a} + \left(\frac{x}{2}\sqrt{16a^2 - x^2} + 8a^2 \sin^{-1}\frac{x}{4a}\right)_{2a}^{4a}\right] \quad 1$$

$$= 2\left[\frac{8\sqrt{3}a^2}{3} + 8a^2\frac{\pi}{2} - 2a^2\sqrt{3} - 8a^2\frac{\pi}{6}\right]$$

$$= 2\left[\frac{2\sqrt{3}a^2}{3} + 8a^2\frac{\pi}{3}\right] \text{sq. units} \quad 1$$

24. Points on the lines are $a_1 = (1, -1, 0)$, $a_2 = (0, 2, -1)$

and the direction of lines is $2\hat{i} - \hat{j} + 3\hat{k}$

let the equation of plane through a_1 be

$a(x-1) + b(y+1) + c(z) = 0$ ...(i) $\quad \frac{1}{2}$

(0, 2, –1) lies on it, $\therefore\ -a + 3b - c = 0$ ...(ii) $\quad 1$

and a, b, c are DR's of a line $\perp$ to the line with DR's 2, –1, 3

$\therefore\ 2a - b + 3c = 0$ ...(iii) $\quad 1$

Solving (ii) & (iii) we get $\frac{a}{8} = \frac{b}{1} = \frac{c}{-5}$ $\quad 1$

$\therefore$ Equation of plane is $8(x - 1) + 1(y + 1) - 5z = 0$

$\Rightarrow\ 8x + y - 5z = 7$ ...(iv) $\quad \frac{1}{2}$

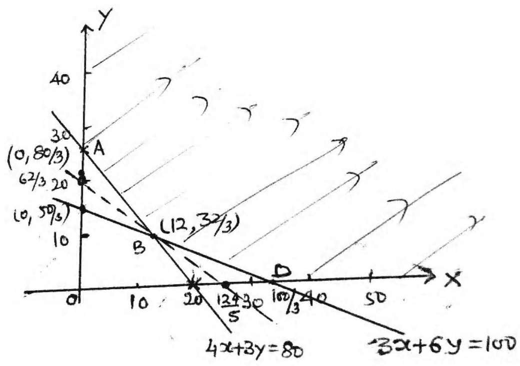
For the line $\frac{x-2}{3} = \frac{y-1}{1} = \frac{z-2}{5}$, since the point (2, 1, 2) lies on plane (iv)

as $8(2) + 1 - 5(2) = 7$ $\quad 1$

and $3(8) + 1(1) + 5(-5) = 25 - 25 = 0$

$\therefore$ The plane (iv) contains the given line $\quad 1$

25. Let x units of F_1 and y units of F_2 be mixed



$\therefore$ We have Minimise cost (C) = $5x + 6y$ 1

subject to
$$\left.\begin{aligned} 4x + 3y &\geq 80 \\ 3x + 6y &\geq 100 \\ x \geq 0,\ y &\geq 0 \end{aligned}\right\}$$ 2

Correct Figure $1\frac{1}{2}$

C(A) = 160

C(B) = 60 + 64 = 124

C(D) = $\frac{500}{3}$

5x + 6y ≤ 124 passes through B only $\frac{1}{2}$

$\therefore$ Minimum cost = ₹ 124

F_1 = 12 units

$F_2 = \frac{32}{3}$ units 1

26. Total number of ways = ${}^6C_3 = 20$ $\frac{1}{2}$

X :	1	2	3	4
P(X) :	$\frac{10}{20}$	$\frac{6}{20}$	$\frac{3}{20}$	$\frac{1}{20}$
XP(X) :	$\frac{10}{20}$	$\frac{12}{20}$	$\frac{9}{20}$	$\frac{4}{20}$
X^2 P(X) :	$\frac{10}{20}$	$\frac{24}{20}$	$\frac{27}{20}$	$\frac{16}{20}$

2

$1\frac{1}{2}$

Mean = $\sum X\,P(X) = \frac{35}{20} = \frac{7}{4}$ 1

Variance = $\sum X^2\,P(X) - \left[\sum X\,P(X)\right]^2 = \frac{77}{20} - \frac{49}{16} = \frac{63}{80}$ 1

QUESTION PAPER CODE 65/2/S

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. Writing or using, that given planes are parallel $\frac{1}{2}$

$d = \frac{|4+10|}{\sqrt{4+9+36}} = 2$ units $\frac{1}{2}$

2. $|\vec{a}-\sqrt{2}\vec{b}|^2 = 1 \Rightarrow \vec{a}.\vec{b} = \frac{1}{\sqrt{2}}.$ $\frac{1}{2}$

$\therefore$ Angle between $\vec{a}$ and $\vec{b}$ $= \frac{\pi}{4}$ $\frac{1}{2}$

3. Getting $\sin\theta = \dfrac{\frac{1}{\sqrt{3}}}{\frac{1}{2}.\frac{4}{\sqrt{3}}} = \frac{1}{2}$ $\frac{1}{2}$

Hence $|\vec{a}.\vec{b}| = \frac{1}{2}.\frac{4}{\sqrt{3}}.\frac{\sqrt{3}}{2} = 1$ $\frac{1}{2}$

4. $k(2) = -8 \Rightarrow k = -4$ $\frac{1}{2}$

$-4(3) = 4a \Rightarrow a = -3$ $\frac{1}{2}$

5. Getting $AB = \begin{pmatrix} 7 & -8 \\ 0 & -10 \end{pmatrix}$ $\frac{1}{2}$

$|AB| = -70$ $\frac{1}{2}$

6. $|AA^T| = |A||A^T| = |A|^2$ $\frac{1}{2}$

$= 25$ $\frac{1}{2}$

SECTION B

7. LHS $= 2\sin^{-1}\left(\frac{3}{5}\right) - \tan^{-1}\left(\frac{17}{31}\right)$

$= 2\tan^{-1}\left(\frac{3}{4}\right) - \tan^{-1}\left(\frac{17}{31}\right)$ 1

$= \tan^{-1}\left(\dfrac{2\cdot\frac{3}{4}}{1-\frac{9}{16}}\right) - \tan^{-1}\left(\frac{17}{31}\right) = \tan^{-1}\left(\frac{24}{7}\right) - \tan^{-1}\left(\frac{17}{31}\right)$ 1

$$\tan^{-1}\left(\frac{\frac{24}{7}-\frac{17}{31}}{1+\frac{24}{7}\cdot\frac{17}{31}}\right)=\tan^{-1}\left(\frac{625}{625}\right)=\tan^{-1}(1)=\frac{\pi}{4}=\text{RHS}$$ 1 + 1

OR

$$\cos\,(\tan^{-1} x) = \sin\left(\cot^{-1}\frac{3}{4}\right)$$

$$\Rightarrow \quad \cos\left(\cos^{-1}\frac{1}{\sqrt{1+x^2}}\right) = \sin\left(\sin^{-1}\frac{4}{5}\right)$$ 1 + 1

$$\Rightarrow \quad \frac{1}{\sqrt{1+x^2}} = \frac{4}{5} \text{ or } \sqrt{1+x^2} = \frac{5}{4}$$ 1

$$\Rightarrow \quad x^2 = \frac{9}{16} \Rightarrow x = \pm\frac{3}{4}$$ 1

8. $I = \int_0^{\pi/2} \frac{\sin^2 x}{\sin x+\cos x}\,dx$ ...(i)

$I = \int_0^{\pi/2} \frac{\sin^2(\pi/2-x)}{\sin(\pi/2-x)+\cos(\pi/2-x)}\,dx = \int_0^{\pi/2} \frac{\cos^2 x}{\cos x+\sin x}\,dx$ ...(ii) 1

$2I = \int_0^{\pi/2} \frac{1}{\sin x+\cos x}\,dx$ 1

$\Rightarrow I = \frac{1}{2\sqrt{2}}\int_0^{\pi/2} \frac{1}{\cos x\frac{1}{\sqrt{2}}+\sin x\frac{1}{\sqrt{2}}}\,dx = \frac{1}{2\sqrt{2}}\int_0^{\pi/2} \sec\left(x-\frac{\pi}{4}\right)dx$ 1

$= \frac{1}{2\sqrt{2}}\left[\log\left|\sec\left(x-\frac{\pi}{4}\right)+\tan\left(x-\frac{\pi}{4}\right)\right|\right]_0^{\pi/2}$ $\frac{1}{2}$

$= \frac{1}{2\sqrt{2}}\log\left|\frac{\sqrt{2}+1}{\sqrt{2}-1}\right|$ or $\frac{1}{\sqrt{2}}\log|\sqrt{2}+1|$ $\frac{1}{2}$

OR

$I = \int_0^1 \cot^{-1}(1-x+x^2)\,dx = \int_0^1 \tan^{-1}\left(\frac{1}{1-x+x^2}\right)dx$ $\frac{1}{2}$

$= \int_0^1 \tan^{-1}\left(\frac{x+(1-x)}{1-x(1-x)}\right)dx = \int_0^1 \tan^{-1} x\,dx + \int_0^1 \tan^{-1}(1-x)\,dx$ 1

$= 2\int_0^1 \tan^{-1} x\,dx$ $\frac{1}{2}$

$= 2\left[\left(\tan^{-1} x.x\right)_0^1 - \int_0^1 \frac{x}{1+x^2}\,dx\right]$ $\frac{1}{2}$

$$= 2\left[x\tan^{-1}x - \frac{1}{2}\log|1+x^2|\right]_0^1$$ 1

$$= 2\left[\frac{\pi}{4} - \frac{1}{2}\log 2\right] \text{ or } \frac{\pi}{2} - \log 2$$ $\frac{1}{2}$

9. $I = \int\left[\log(\log x) + \frac{1}{(\log x)^2}\right]dx$

$$= \int \log(\log x).1\,dx + \int \frac{1}{(\log x)^2}\,dx$$ 1

$$= \log(\log x)\cdot x - \int \frac{1}{\log x}\cdot\frac{1}{\cancel{x}}\cdot\cancel{x}\,dx + \int \frac{1}{(\log x)^2}\,dx$$ 2

$$= x\cdot\log(\log x) - \left[\frac{1}{\log x}\cdot x - \int \frac{-1}{(\log x)^2}\cdot\frac{1}{x}\cdot x\,dx\right] + \int \frac{1}{(\log x)^2}\,dx$$ $\frac{1}{2}$

$$= x\log(\log x) - \frac{x}{\log x} + C$$ $\frac{1}{2}$

10. Writing $\int \frac{1-\sin x}{\sin x(1+\sin x)}\,dx = \int \frac{(1+\sin x)-2\sin x}{\sin x(1+\sin x)}\,dx$ 1

$$= \int \frac{1}{\sin x}\,dx - 2\int \frac{1}{1+\sin x}\,dx$$ 1

$$= \int \operatorname{cosec} x\,dx - 2\int \frac{(1-\sin x)}{\cos^2 x}\,dx$$ 1

$$= \log|\operatorname{cosec} x - \cot x| - 2\int (\sec^2 x - \sec x\tan x)\,dx$$ $\frac{1}{2}$

$$= \log|\operatorname{cosec} x - \cot x| - 2(\tan x - \sec x) + C$$ $\frac{1}{2}$

11. When $x = am^2$, we get $y = \pm am^3$ 1

$$ay^2 = x^3 \Rightarrow 2ay\frac{dy}{dx} = 3x^2 \Rightarrow \frac{dy}{dx} = \frac{3x^2}{2ay}$$ 1

$$\text{slope of normal} = \mp\frac{2a}{3}\,\frac{am^3}{a^2m^4} = \mp\frac{2}{3m}$$ 1

$\therefore$ Equation of normal is $y \mp am^3 = \mp\frac{2}{3m}(x - am^2)$ 1

[Full marks may be given, if only one value for point, slope and equation is derived]

12. $\text{LHL} = \lim_{x\to 0^-} k.\sin\frac{\pi}{2}(x+1) = k$ 1

$$\text{RHL} = \lim_{x\to 0^+} \frac{\tan x(1-\cos x)}{x^3}$$ 1

$$= \lim_{x \to 0^+} \frac{\tan x}{x} . 2\left(\frac{\sin x/2}{2x/2}\right)^2 = \frac{1}{2}$$ 1

$$\Rightarrow \quad k = \frac{1}{2}$$ 1

13. $y = (\sin 2x)^x + \sin^{-1}(\sqrt{3x}) = u + v$

$$\therefore \quad \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$ $\frac{1}{2}$

$$u = (\sin 2x)^x \Rightarrow \log u = x \log \sin 2x$$ $\frac{1}{2}$

$$\frac{1}{u}\frac{du}{dx} = 2x \cdot \cot 2x + \log \sin 2x$$ 1

$$\therefore \quad \frac{du}{dx} = (\sin 2x)^x [2x \cot 2x + \log \sin 2x]$$ $\frac{1}{2}$

$$\frac{dv}{dx} = \frac{1}{\sqrt{1-3x}} \frac{\sqrt{3}}{2\sqrt{x}}$$ 1

$$\therefore \quad \frac{dy}{dx} = (\sin 2x)^x [2x \cot 2x + \log \sin 2x] + \frac{\sqrt{3}}{2\sqrt{x}\sqrt{1-3x}}$$ $\frac{1}{2}$

OR

Let $y = \tan^{-1}\left(\frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}}\right)$ and $z = \cos^{-1} x^2$

$$z = \cos^{-1} x^2 \Rightarrow x^2 = \cos z \Rightarrow y = \tan^{-1}\left(\frac{\sqrt{1+\cos z} - \sqrt{1-\cos z}}{\sqrt{1+\cos z} + \sqrt{1-\cos z}}\right)$$ 1

$$\therefore \quad y = \tan^{-1}\left(\frac{\cos\frac{z}{2} - \sin\frac{z}{2}}{\cos\frac{z}{2} + \sin\frac{z}{2}}\right) = \tan^{-1}\left(\frac{1-\tan\frac{z}{2}}{1+\tan\frac{z}{2}}\right)$$ $\frac{1}{2} + \frac{1}{2}$

$$\therefore \quad y = \tan^{-1}\left[\tan\left(\frac{\pi}{4} - \frac{z}{2}\right)\right] = \frac{\pi}{4} - \frac{z}{2}$$ $\frac{1}{2} + \frac{1}{2}$

$$\Rightarrow \quad \frac{dy}{dz} = -\frac{1}{2}$$ 1

14. Equation of plane passing through A, B and C is

$$\begin{vmatrix} x-3 & y-2 & z-1 \\ 1 & 0 & -3 \\ 2 & 3 & 1 \end{vmatrix} = 0$$ 2

$\Rightarrow (x - 3)9 - (y - 2)7 + (z - 1)3 = 0 \Rightarrow 9x - 7y + 3z = 16 \quad \ldots$(i) 1

If A, B, C and D are coplanar, D must lie on (i)

$\Rightarrow 9\lambda - 35 + 15 - 16 = 0 \Rightarrow \lambda = 4.$ 1

OR

Equation of plane, perpendicular to $\vec{n} = \hat{i} + \hat{j} + 3\hat{k}$ and at a distance $\frac{4}{\sqrt{11}}$ from origin is

$\vec{r} \cdot \frac{(\hat{i} + \hat{j} + 3\hat{k})}{\sqrt{11}} = \frac{4}{\sqrt{11}}$ or $\vec{r} \cdot (\hat{i} + \hat{j} + 3\hat{k}) = 4$ ...(i) $1\frac{1}{2}$

Any point on the line $\vec{r} = (-\hat{i} - 2\hat{j} - 3\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 3\hat{k})$ is

$(-1+3\lambda)\hat{i} + (-2+4\lambda)\hat{j} + (-3+3\lambda)\hat{k}$ ...(ii) 1

If this point is the point of intersection of the plane and the line then,

$(-1+3\lambda)1 + (-2+4\lambda)1 + (-3+3\lambda)3 = 4$

$\Rightarrow \lambda = 1.$ 1

Hence the point of intersection is (2, 2, 0) $\frac{1}{2}$

15. $\vec{a} = \vec{b} + \vec{c} \Rightarrow p\hat{i} + q\hat{j} + r\hat{k} = (s+3)\hat{i} + 4\hat{j} + 2\hat{k}$

$p = s + 3,\ q = 4,\ r = 2$ $1\frac{1}{2}$

area $= \frac{1}{2}|\vec{b} \times \vec{c}| = 5\sqrt{6}$

$$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ s & 3 & 4 \\ 3 & 1 & -2 \end{vmatrix} = -10\hat{i} + (2s+12)\hat{j} + (s-9)\hat{k}$$ $\frac{1}{2}$

$\therefore\ 100 + (2s + 12)^2 + (s - 9)^2 = (10\sqrt{6})^2 = 600$

$\Rightarrow s^2 + 6s + 55 = 0 \Rightarrow s = -11,\ p = -8,$ or $s = 5,\ p = 8$ 1 + 1

16. Let E_1 : selecting bag A, E_2 : selecting bag B $\frac{1}{2}$

A : getting 2 white and 1 red out of 3 drawn (without replacement)

$\therefore\ P(E_1) = P(E_2) = \frac{1}{2}$ $\frac{1}{2}$

$P\left(\frac{A}{E_1}\right) = \frac{^3C_2 \cdot {^4C_1}}{^7C_3} = \frac{12}{35}$ 1

$P\left(\frac{A}{E_2}\right) = \frac{^4C_2 \cdot {^3C_1}}{^7C_3} = \frac{18}{35}$ 1

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)\, P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{2}\cdot\frac{18}{35}}{\frac{1}{2}\cdot\frac{12}{35} + \frac{1}{2}\cdot\frac{18}{35}} = \frac{3}{5}$$ 1

17. Let length be x m and breadth be y m

$\therefore \quad (x-50)(y+50) = xy \Rightarrow 50x - 50y = 2500$ or $x - y = 50$ $\quad \frac{1}{2}$

and $\quad (x-10)(y-20) = xy - 5300 \Rightarrow 2x + y = 550$ $\quad \frac{1}{2}$

$$\begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 50 \\ 550 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 & 1 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 50 \\ 550 \end{pmatrix} \qquad \frac{1}{2} + 1$$

$\Rightarrow \quad x = \frac{1}{3}(600) = 200$ m, $\; y = \frac{1}{3}(450) = 150$ m $\quad \frac{1}{2}$

"Helping the children of his village to learn" (or any other relavent value) $\quad 1$

18. From the given differential equation, we can write

$$\frac{dx}{dy} = \frac{2xe^{x/y} - y}{2ye^{x/y}} = \frac{2\,x/y\;e^{x/y} - 1}{2e^{x/y}} \qquad 1$$

Putting $\frac{x}{y} = v \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$ $\quad \frac{1}{2}$

$$\therefore \quad v + y\frac{dv}{dy} = \frac{2ve^v - 1}{2e^v} \Rightarrow y\frac{dv}{dy} = -\frac{1}{2e^v} \qquad 1$$

$$\Rightarrow \quad 2\int e^v\, dv = -\int \frac{dy}{y} \qquad \frac{1}{2}$$

$$\therefore \quad 2e^v + \log|y| = C \Rightarrow 2e^{x/y} + \log|y| = C \qquad 1$$

19. The given differential equation can be written as

$$\frac{dy}{dx} - \frac{1}{x+1}\,y = (x+1)^2.e^{3x} \qquad \frac{1}{2}$$

Here, integrating factor $= e^{\int -\frac{1}{x+1}dx} = \frac{1}{x+1}$ $\quad 1$

$\therefore \quad$ Solution is $\; y\frac{1}{x+1} = \int (x+1)\,e^{3x}dx$ $\quad 1$

$$\therefore \quad \frac{y}{x+1} = (x+1)\frac{e^{3x}}{3} - \frac{e^{3x}}{9} + C \qquad 1\frac{1}{2}$$

or $\quad y = \left[\frac{1}{3}(x+1)^2 - \frac{x+1}{9}\right]e^{3x} + C(x+1)$

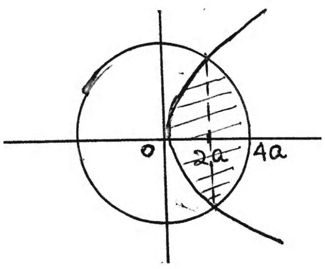
SECTION C

20. Solving $y^2 = 6ax$ and $x^2 + y^2 = 16a^2$

we get $x^2 + 6ax - 16a^2 = 0$

$(x + 8a)(x - 2a) = 0$

$x = -8a,\ x = 2a$ 1



Correct Figure 1

Required area $= 2\left[\int_0^{2a} \sqrt{6a}\sqrt{x}\, dx + \int_{2a}^{4a} \sqrt{16a^2 - x^2}\, dx\right]$ 2

$= 2\left[\left(\sqrt{6}\sqrt{a}\,\frac{2}{3}x^{3/2}\right)_0^{2a} + \left(\frac{x}{2}\sqrt{16a^2 - x^2} + 8a^2 \sin^{-1}\frac{x}{4a}\right)_{2a}^{4a}\right]$ 1

$= 2\left[\frac{8\sqrt{3}a^2}{3} + 8a^2\frac{\pi}{2} - 2a^2\sqrt{3} - 8a^2\frac{\pi}{6}\right]$

$= 2\left[\frac{2\sqrt{3}a^2}{3} + 8a^2\frac{\pi}{3}\right]$ sq. units 1

21. $f'(x) = 4x^3 - 24x^2 + 44x - 24$ 1

$= 4(x^3 - 6x^2 + 11x - 6) = 4(x - 1)(x - 2)(x - 3)$ $1\frac{1}{2}$

$f'(x) = 0 \Rightarrow x = 1,\ x = 2,\ x = 3$ $\frac{1}{2}$

The intervals are $(-\infty, 1), (1, 2), (2, 3), (3, \infty)$ 1

since $f'(x) > 0$ in $(1, 2)$ and $(3, \infty)$

$\therefore$ $f(x)$ is strictly increasing in $(1, 2) \cup (3, \infty)$ 1

and strictly decreasing in $(-\infty, 1) \cup (2, 3)$ 1

OR

$f(x) = \sec x + 2 \log |\cos x|$

$f'(x) = \sec x \tan x - 2 \tan x = \tan x (\sec x - 2)$ 1

$f'(x) = 0 \Rightarrow \tan x = 0$ or $\sec x = 2$

$\Rightarrow x = \pi,\ x = \frac{\pi}{3}, \frac{5\pi}{3}$ $1\frac{1}{2}$

$f''(x) = \sec x \tan^2 x + (\sec x - 2) \sec^2 x$ 1

$$\left.\begin{array}{l} f''(\pi/3) = 6\ (+\text{ve}) \Rightarrow f(x) \text{ is minimum at } x = \pi/3 \\ f''(\pi) = -3\ (-\text{ve}) \Rightarrow f(x) \text{ is maximum at } x = \pi \\ f''(5\pi/3) = 6\ (+\text{ve}) \Rightarrow f(x) \text{ is minimum at } x = 5\pi/3 \end{array}\right\}\ 1\frac{1}{2}$$

Maximum value $= f(\pi) = -1.$ $\frac{1}{2}$

Minimum value $= f(\pi/3) = f(5\pi/3) = 2 - 2 \log 2$ or $2 + \log (1/4)$ $\frac{1}{2}$

22. Let $\Delta = \begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix}$

$C_1 \to C_1 + C_2 - 2C_3 \Rightarrow \Delta = (a^2+b^2+c^2)\begin{vmatrix} 1 & a^2 & bc \\ 1 & b^2 & ca \\ 1 & c^2 & ab \end{vmatrix}$ 1

$R_1 \to R_1 - R_2$, and $R_2 \to R_2 - R_3 \Rightarrow \Delta = (a^2+b^2+c^2)\begin{vmatrix} 0 & a^2-b^2 & c(b-a) \\ 0 & b^2-c^2 & a(c-b) \\ 1 & c^2 & ab \end{vmatrix}$ $1\frac{1}{2}$

$= (a^2+b^2+c^2)(a-b)(b-c)\begin{vmatrix} 0 & a+b & -c \\ 0 & b+c & -a \\ 1 & c^2 & ab \end{vmatrix}$ $\frac{1}{2}$

$R_2 \to R_2 - R_1 \Rightarrow \Delta = (a^2+b^2+c^2)(a-b)(b-c)\begin{vmatrix} 0 & a+b & -c \\ 0 & c-a & c-a \\ 1 & c^2 & ab \end{vmatrix}$ 1

$\therefore\ \Delta = (a^2+b^2+c^2)(a-b)(b-c)(c-a)\begin{vmatrix} 0 & a+b & -c \\ 0 & 1 & 1 \\ 1 & c^2 & ab \end{vmatrix}$ 1

Expanding by C_1 to get $\Delta = (a^2+b^2+c^2)(a-b)(b-c)(c-a)(a+b+c)$ 1

OR

Let A = IA $\therefore \begin{pmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$ 1

$R_2 \to R_2 - 2R_3 \Rightarrow \begin{pmatrix} 2 & -1 & 3 \\ 1 & -1 & -5 \\ -3 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix} A$ $\frac{1}{2}$

$\begin{matrix} R_1 \to R_1 - 2R_2 \\ R_3 \to R_3 + 3R_2 \end{matrix} \Rightarrow \begin{pmatrix} 0 & 1 & 13 \\ 1 & -1 & -5 \\ 0 & -1 & -12 \end{pmatrix} = \begin{pmatrix} 1 & -2 & 4 \\ 0 & 1 & -2 \\ 0 & 3 & -5 \end{pmatrix} A$ 1

$R_1 \leftrightarrow R_2 \Rightarrow \begin{pmatrix} 1 & -1 & -5 \\ 0 & 1 & 13 \\ 0 & -1 & -12 \end{pmatrix} = \begin{pmatrix} 0 & 1 & -2 \\ 1 & -2 & 4 \\ 0 & 3 & -5 \end{pmatrix} A$ $\frac{1}{2}$

$\begin{cases} R_1 \to R_1 + R_2 \\ R_3 \to R_3 + R_2 \end{cases} \Rightarrow \begin{pmatrix} 1 & 0 & 8 \\ 0 & 1 & 13 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 2 \\ 1 & -2 & 4 \\ 1 & 1 & -1 \end{pmatrix} A$ 1

$$\begin{cases} R_1 \to R_1 - 8R_3 \\ R_2 \to R_2 - 13R_3 \end{cases} \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -7 & -9 & 10 \\ -12 & -15 & 17 \\ 1 & 1 & -1 \end{pmatrix} A$$ 1

$$\Rightarrow A^{-1} = \begin{pmatrix} -7 & -9 & 10 \\ -12 & -15 & 17 \\ 1 & 1 & -1 \end{pmatrix}$$ 1

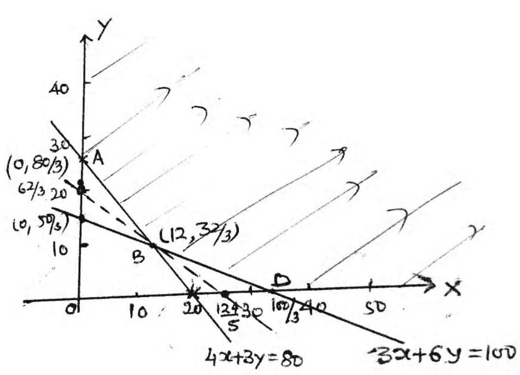
23. Total number of ways = ${}^6C_3 = 20$ $\frac{1}{2}$

X :	1	2	3	4	2
P(X) :	$\frac{10}{20}$	$\frac{6}{20}$	$\frac{3}{20}$	$\frac{1}{20}$	$1\frac{1}{2}$
XP(X) :	$\frac{10}{20}$	$\frac{12}{20}$	$\frac{9}{20}$	$\frac{4}{20}$	
X^2 P(X) :	$\frac{10}{20}$	$\frac{24}{20}$	$\frac{27}{20}$	$\frac{16}{20}$	

Mean = $\sum X\,P(X) = \frac{35}{20} = \frac{7}{4}$ 1

Variance = $\sum X^2\,P(X) - \left[\sum X\,P(X)\right]^2 = \frac{77}{20} - \frac{49}{16} = \frac{63}{80}$ 1

24.



Let x units of F_1 and y units of F_2 be mixed

$\therefore$ We have Minimise cost (C) = $5x + 6y$ 1

subject to
$$\left.\begin{aligned} 4x + 3y &\geq 80 \\ 3x + 6y &\geq 100 \\ x \geq 0,\ y &\geq 0 \end{aligned}\right\}$$ 2

Correct Figure $1\frac{1}{2}$

C(A) = 160

C(B) = 60 + 64 = 124

C(D) = $\frac{500}{3}$

5x + 6y ≤ 124 passes through B only $\frac{1}{2}$

$\therefore$ Minimum cost = ₹ 124

F_1 = 12 units

$F_2 = \frac{32}{3}$ units 1

25. Points on the lines are $a_1 = (1, -1, 0)$, $a_2 = (0, 2, -1)$

and the direction of lines is $2\hat{i} - \hat{j} + 3\hat{k}$

let the equation of plane through a_1 be

$a(x-1) + b(y+1) + c(z) = 0$ ...(i) $\frac{1}{2}$

(0, 2, –1) lies on it, $\therefore -a + 3b - c = 0$ ...(ii) 1

and a, b, c are DR's of a line $\perp$ to the line with DR's 2, –1, 3

$\therefore 2a - b + 3c = 0$ ...(iii) 1

Solving (ii) & (iii) we get $\frac{a}{8} = \frac{b}{1} = \frac{c}{-5}$ 1

$\therefore$ Equation of plane is $8(x - 1) + 1(y + 1) - 5z = 0$

$\Rightarrow 8x + y - 5z = 7$ ...(iv) $\frac{1}{2}$

For the line $\frac{x-2}{3} = \frac{y-1}{1} = \frac{z-2}{5}$, since the point (2, 1, 2) lies on plane (iv)

as $8(2) + 1 - 5(2) = 7$ 1

and $3(8) + 1(1) + 5(-5) = 25 - 25 = 0$

$\therefore$ The plane (iv) contains the given line 1

26. Let $x_1, x_2 \in \text{N}$ and $f(x_1) = f(x_2)$

$\Rightarrow 4x_1^2 + 12x_1 + 15 = 4x_2^2 + 12x_2 + 15$

$\Rightarrow 4(x_1^2 - x_2^2) + 12(x_1 - x_2) = 0 \Rightarrow (x_1 - x_2)(4x_1 + 4x_2 + 12) = 0$

$\Rightarrow x_1 - x_2 = 0$ or $x_1 = x_2$ as $4x_1 + 4x_2 + 12 \neq 0$, $x_1, x_2 \in \text{N}$

$\therefore f$ is a 1 – 1 function

2

$f : \text{N} \to \text{S}$ is onto as co-domain = range 1

Hence f is invertible.

$$y = 4x^2 + 12x + 15 = (2x+3)^2 + 6 \Rightarrow x = \frac{\sqrt{y-6} - 3}{2}$$

$\therefore \quad f^{-1}(y) = \frac{\sqrt{y-6} - 3}{2}, \; y \in \text{S}.$ 2

$f^{-1}(31) = \frac{\sqrt{31-6} - 3}{2} = 1$ $\frac{1}{2}$

$f^{-1}(87) = \frac{\sqrt{87-6} - 3}{2} = 3$ $\frac{1}{2}$

QUESTION PAPER CODE 65/3/S

EXPECTED ANSWER/VALUE POINTS

SECTION A

1. $|AA^T| = |A||A^T| = |A|^2$ $\frac{1}{2}$

$= 25$ $\frac{1}{2}$

2. Writing or using, that given planes are parallel $\frac{1}{2}$

$d = \frac{|4+10|}{\sqrt{4+9+36}} = 2$ units $\frac{1}{2}$

3. $|\vec{a} - \sqrt{2}\vec{b}|^2 = 1 \Rightarrow \vec{a}.\vec{b} = \frac{1}{\sqrt{2}}.$ $\frac{1}{2}$

$\therefore$ Angle between $\vec{a}$ and $\vec{b}$ $= \frac{\pi}{4}$ $\frac{1}{2}$

4. Getting AB = $\begin{pmatrix} 7 & -8 \\ 0 & -10 \end{pmatrix}$ $\frac{1}{2}$

$|AB| = -70$ $\frac{1}{2}$

5. $k(2) = -8 \Rightarrow k = -4$ $\frac{1}{2}$

$-4(3) = 4a \Rightarrow a = -3$ $\frac{1}{2}$

6. Getting $\sin\theta = \frac{\frac{1}{\sqrt{3}}}{\frac{1}{2}.\frac{4}{\sqrt{3}}} = \frac{1}{2}$ $\frac{1}{2}$

Hence $|\vec{a}.\vec{b}| = \frac{1}{2}.\frac{4}{\sqrt{3}}.\frac{\sqrt{3}}{2} = 1$ $\frac{1}{2}$

SECTION B

7. LHL = $\lim_{x \to 0^-} k.\sin\frac{\pi}{2}(x+1) = k$ 1

RHL = $\lim_{x \to 0^+} \frac{\tan x(1-\cos x)}{x^3}$ 1

$= \lim_{x \to 0^+} \frac{\tan x}{x}.2\left(\frac{\sin x/2}{2x/2}\right)^2 = \frac{1}{2}$ 1

$\Rightarrow \quad k = \frac{1}{2}$ 1

8. $y = (\sin 2x)^x + \sin^{-1}(\sqrt{3x}) = u + v$

$\therefore \quad \dfrac{dy}{dx} = \dfrac{du}{dx} + \dfrac{dv}{dx}$ $\quad \dfrac{1}{2}$

$u = (\sin 2x)^x \Rightarrow \log u = x \log \sin 2x$ $\quad \dfrac{1}{2}$

$\dfrac{1}{u}\dfrac{du}{dx} = 2x \cdot \cot 2x + \log \sin 2x$ $\quad 1$

$\therefore \quad \dfrac{du}{dx} = (\sin 2x)^x [2x \cot 2x + \log \sin 2x]$ $\quad \dfrac{1}{2}$

$\dfrac{dv}{dx} = \dfrac{1}{\sqrt{1-3x}} \dfrac{\sqrt{3}}{2\sqrt{x}}$ $\quad 1$

$\therefore \quad \dfrac{dy}{dx} = (\sin 2x)^x [2x \cot 2x + \log \sin 2x] + \dfrac{\sqrt{3}}{2\sqrt{x}\sqrt{1-3x}}$ $\quad \dfrac{1}{2}$

OR

Let $y = \tan^{-1}\left(\dfrac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}}\right)$ and $z = \cos^{-1} x^2$

$z = \cos^{-1} x^2 \Rightarrow x^2 = \cos z \Rightarrow y = \tan^{-1}\left(\dfrac{\sqrt{1+\cos z} - \sqrt{1-\cos z}}{\sqrt{1+\cos z} + \sqrt{1-\cos z}}\right)$ $\quad 1$

$\therefore \quad y = \tan^{-1}\left(\dfrac{\cos\frac{z}{2} - \sin\frac{z}{2}}{\cos\frac{z}{2} + \sin\frac{z}{2}}\right) = \tan^{-1}\left(\dfrac{1-\tan\frac{z}{2}}{1+\tan\frac{z}{2}}\right)$ $\quad \dfrac{1}{2} + \dfrac{1}{2}$

$\therefore \quad y = \tan^{-1}\left[\tan\left(\dfrac{\pi}{4} - \dfrac{z}{2}\right)\right] = \dfrac{\pi}{4} - \dfrac{z}{2}$ $\quad \dfrac{1}{2} + \dfrac{1}{2}$

$\Rightarrow \quad \dfrac{dy}{dz} = -\dfrac{1}{2}$ $\quad 1$

9. $\vec{a} = \vec{b} + \vec{c} \Rightarrow p\hat{i} + q\hat{j} + r\hat{k} = (s+3)\hat{i} + 4\hat{j} + 2\hat{k}$

$p = s + 3, \ q = 4, \ r = 2$ $\quad 1\dfrac{1}{2}$

area $= \dfrac{1}{2}|\vec{b} \times \vec{c}| = 5\sqrt{6}$

$\vec{b} \times \vec{c} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ s & 3 & 4 \\ 3 & 1 & -2 \end{vmatrix} = -10\hat{i} + (2s+12)\hat{j} + (s-9)\hat{k}$ $\quad \dfrac{1}{2}$

$\therefore \ 100 + (2s + 12)^2 + (s - 9)^2 = (10\sqrt{6})^2 = 600$

$\Rightarrow s^2 + 6s + 55 = 0 \Rightarrow s = -11, \ p = -8, \text{ or } s = 5, \ p = 8$ $\quad 1 + 1$

10. Let E_1 : selecting bag A, E_2 : selecting bag B $\frac{1}{2}$

A : getting 2 white and 1 red out of 3 drawn (without replacement)

$\therefore$ $P(E_1) = P(E_2) = \frac{1}{2}$ $\frac{1}{2}$

$P\left(\frac{A}{E_1}\right) = \frac{{}^3C_2 \cdot {}^4C_1}{{}^7C_3} = \frac{12}{35}$ 1

$P\left(\frac{A}{E_2}\right) = \frac{{}^4C_2 \cdot {}^3C_1}{{}^7C_3} = \frac{18}{35}$ 1

$$P\left(\frac{E_2}{A}\right) = \frac{P(E_2)\,P\left(\frac{A}{E_2}\right)}{P(E_1)P\left(\frac{A}{E_1}\right) + P(E_2)P\left(\frac{A}{E_2}\right)}$$

$$= \frac{\frac{1}{2}\cdot\frac{18}{35}}{\frac{1}{2}\cdot\frac{12}{35} + \frac{1}{2}\cdot\frac{18}{35}} = \frac{3}{5}$$ 1

11. Let length be x m and breadth be y m

$\therefore$ $(x - 50)(y + 50) = xy \Rightarrow 50x - 50y = 2500$ or $x - y = 50$ $\frac{1}{2}$

and $(x - 10)(y - 20) = xy - 5300 \Rightarrow 2x + y = 550$ $\frac{1}{2}$

$\begin{pmatrix} 1 & -1 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 50 \\ 550 \end{pmatrix} \Rightarrow \begin{pmatrix} x \\ y \end{pmatrix} = \frac{1}{3}\begin{pmatrix} 1 & 1 \\ -2 & 1 \end{pmatrix}\begin{pmatrix} 50 \\ 550 \end{pmatrix}$ $\frac{1}{2} + 1$

$\Rightarrow$ $x = \frac{1}{3}(600) = 200$ m, $y = \frac{1}{3}(450) = 150$ m $\frac{1}{2}$

"Helping the children of his village to learn" (or any other relavent value) 1

12. From the given differential equation, we can write

$\frac{dx}{dy} = \frac{2xe^{x/y} - y}{2ye^{x/y}} = \frac{2\,x/y\,e^{x/y} - 1}{2e^{x/y}}$ 1

Putting $\frac{x}{y} = v \Rightarrow \frac{dx}{dy} = v + y\frac{dv}{dy}$ $\frac{1}{2}$

$\therefore$ $v + y\frac{dv}{dy} = \frac{2ve^v - 1}{2e^v} \Rightarrow y\frac{dv}{dy} = -\frac{1}{2e^v}$ 1

$\Rightarrow$ $2\int e^v\,dv = -\int\frac{dy}{y}$ $\frac{1}{2}$

$\therefore$ $2e^v + \log|y| = C \Rightarrow 2e^{x/y} + \log|y| = C$ 1

13. The given differential equation can be written as

$$\frac{dy}{dx}-\frac{1}{x+1}\,y = (x+1)^2.e^{3x}$$ $\frac{1}{2}$

Here, integrating factor = $e^{\int -\frac{1}{x+1}dx} = \frac{1}{x+1}$ 1

$\therefore$ Solution is $y\frac{1}{x+1}=\int (x+1)\,e^{3x}dx$ 1

$\therefore$ $\frac{y}{x+1} = (x+1)\frac{e^{3x}}{3}-\frac{e^{3x}}{9}+C$ $1\frac{1}{2}$

or $y = \left[\frac{1}{3}(x+1)^2-\frac{x+1}{9}\right]e^{3x}+C(x+1)$

14. $I = \int\left[\log(\log x)+\frac{1}{(\log x)^2}\right]dx$

$= \int \log(\log x).1\,dx+\int \frac{1}{(\log x)^2}\,dx$ 1

$= \log(\log x)\cdot x-\int \frac{1}{\log x}\cdot\frac{1}{\cancel{x}}\cdot\cancel{x}\,dx+\int\frac{1}{(\log x)^2}dx$ 2

$= x\cdot\log(\log x)-\left[\frac{1}{\log x}\cdot x-\int\frac{-1}{(\log x)^2}\cdot\frac{1}{x}\cdot x\,dx\right]+\int\frac{1}{(\log x)^2}\,dx$ $\frac{1}{2}$

$= x\log(\log x)-\frac{x}{\log x}+C$ $\frac{1}{2}$

15. LHS $= 2\sin^{-1}\left(\frac{3}{5}\right)-\tan^{-1}\left(\frac{17}{31}\right)$

$= 2\tan^{-1}\left(\frac{3}{4}\right)-\tan^{-1}\left(\frac{17}{31}\right)$ 1

$= \tan^{-1}\left(\frac{2\cdot\frac{3}{4}}{1-\frac{9}{16}}\right)-\tan^{-1}\left(\frac{17}{31}\right)=\tan^{-1}\left(\frac{24}{7}\right)-\tan^{-1}\left(\frac{17}{31}\right)$ 1

$\tan^{-1}\left(\frac{\frac{24}{7}-\frac{17}{31}}{1+\frac{24}{7}\cdot\frac{17}{31}}\right)=\tan^{-1}\left(\frac{625}{625}\right)=\tan^{-1}(1)=\frac{\pi}{4}=\text{RHS}$ 1 + 1

OR

$$\cos\left(\tan^{-1} x\right) = \sin\left(\cot^{-1}\frac{3}{4}\right)$$

$$\Rightarrow \quad \cos\left(\cos^{-1}\frac{1}{\sqrt{1+x^2}}\right) = \sin\left(\sin^{-1}\frac{4}{5}\right)$$ 1 + 1

$$\Rightarrow \quad \frac{1}{\sqrt{1+x^2}} = \frac{4}{5} \text{ or } \sqrt{1+x^2} = \frac{5}{4}$$ 1

$$\Rightarrow \quad x^2 = \frac{9}{16} \Rightarrow x = \pm\frac{3}{4}$$ 1

16. Writing $\int \frac{1-\sin x}{\sin x(1+\sin x)}\,dx = \int \frac{(1+\sin x)-2\sin x}{\sin x(1+\sin x)}\,dx$ 1

$$= \int \frac{1}{\sin x}\,dx - 2\int \frac{1}{1+\sin x}\,dx$$ 1

$$= \int \operatorname{cosec} x\,dx - 2\int \frac{(1-\sin x)}{\cos^2 x}\,dx$$ 1

$$= \log|\operatorname{cosec} x - \cot x| - 2\int (\sec^2 x - \sec x \tan x)\,dx$$ $\frac{1}{2}$

$$= \log|\operatorname{cosec} x - \cot x| - 2(\tan x - \sec x) + C$$ $\frac{1}{2}$

17. $I = \int_0^{\pi/2} \frac{\sin^2 x}{\sin x + \cos x}\,dx$ ...(i)

$$I = \int_0^{\pi/2} \frac{\sin^2(\pi/2 - x)}{\sin(\pi/2 - x) + \cos(\pi/2 - x)}\,dx = \int_0^{\pi/2} \frac{\cos^2 x}{\cos x + \sin x}\,dx \quad \text{...(ii)}$$ 1

$$2I = \int_0^{\pi/2} \frac{1}{\sin x + \cos x}\,dx$$ 1

$$\Rightarrow I = \frac{1}{2\sqrt{2}}\int_0^{\pi/2} \frac{1}{\cos x\frac{1}{\sqrt{2}} + \sin x\frac{1}{\sqrt{2}}}\,dx = \frac{1}{2\sqrt{2}}\int_0^{\pi/2} \sec\left(x - \frac{\pi}{4}\right)dx$$ 1

$$= \frac{1}{2\sqrt{2}}\left[\log\left|\sec\left(x-\frac{\pi}{4}\right) + \tan\left(x-\frac{\pi}{4}\right)\right|\right]_0^{\pi/2}$$ $\frac{1}{2}$

$$= \frac{1}{2\sqrt{2}}\log\left|\frac{\sqrt{2}+1}{\sqrt{2}-1}\right| \text{ or } \frac{1}{\sqrt{2}}\log|\sqrt{2}+1|$$ $\frac{1}{2}$

OR

$$I = \int_0^1 \cot^{-1}(1-x+x^2)\,dx = \int_0^1 \tan^{-1}\left(\frac{1}{1-x+x^2}\right)dx$$ $\frac{1}{2}$

$$= \int_0^1 \tan^{-1}\left(\frac{x+(1-x)}{1-x(1-x)}\right)dx = \int_0^1 \tan^{-1} x\,dx + \int_0^1 \tan^{-1}(1-x)\,dx$$ 1

$$= 2\int_0^1 \tan^{-1} x\,dx$$ $\frac{1}{2}$

$$= 2\left[\left(\tan^{-1} x . x\right)_0^1 - \int_0^1 \frac{x}{1+x^2}\,dx\right]$$ $\frac{1}{2}$

$$= 2\left[x\tan^{-1} x - \frac{1}{2}\log|1+x^2|\right]_0^1$$ 1

$$= 2\left[\frac{\pi}{4} - \frac{1}{2}\log 2\right] \text{ or } \frac{\pi}{2} - \log 2$$ $\frac{1}{2}$

18. When $x = am^2$, we get $y = \pm am^3$ 1

$$ay^2 = x^3 \Rightarrow 2ay\frac{dy}{dx} = 3x^2 \Rightarrow \frac{dy}{dx} = \frac{3x^2}{2ay}$$ 1

slope of normal = $\mp\frac{2a}{3}\frac{am^3}{a^2m^4} = \mp\frac{2}{3m}$ 1

$\therefore$ Equation of normal is $y \mp am^3 = \mp\frac{2}{3m}(x - am^2)$ 1

[Full marks may be given, if only one value for point, slope and equation is derived]

19. Equation of plane passing through A, B and C is

$$\begin{vmatrix} x-3 & y-2 & z-1 \\ 1 & 0 & -3 \\ 2 & 3 & 1 \end{vmatrix} = 0$$

2

$\Rightarrow (x-3)9 - (y-2)7 + (z-1)3 = 0 \Rightarrow 9x - 7y + 3z = 16$ ...(i)
1

If A, B, C and D are coplanar, D must lie on (i)

$\Rightarrow 9\lambda - 35 + 15 - 16 = 0 \Rightarrow \lambda = 4.$ 1

OR

Equation of plane, perpendicular to $\vec{n} = \hat{i} + \hat{j} + 3\hat{k}$ and at a distance $\frac{4}{\sqrt{11}}$ from origin is

$\vec{r}\cdot\frac{(\hat{i}+\hat{j}+3\hat{k})}{\sqrt{11}} = \frac{4}{\sqrt{11}}$ or $\vec{r}\cdot(\hat{i}+\hat{j}+3\hat{k}) = 4$ ...(i) $1\frac{1}{2}$

Any point on the line $\vec{r} = (-\hat{i} - 2\hat{j} - 3\hat{k}) + \lambda(3\hat{i} + 4\hat{j} + 3\hat{k})$ is

$(-1+3\lambda)\hat{i} + (-2+4\lambda)\hat{j} + (-3+3\lambda)\hat{k}$ ...(ii) 1

If this point is the point of intersection of the plane and the line then,

$(-1+3\lambda)1+(-2+4\lambda)1+(-3+3\lambda)3=4$

$\Rightarrow \lambda = 1.$ 1

Hence the point of intersection is (2, 2, 0) $\frac{1}{2}$

SECTION C

20. Total number of ways = ${}^6C_3 = 20$ $\frac{1}{2}$

X :	1	2	3	4
P(X) :	$\frac{10}{20}$	$\frac{6}{20}$	$\frac{3}{20}$	$\frac{1}{20}$
XP(X) :	$\frac{10}{20}$	$\frac{12}{20}$	$\frac{9}{20}$	$\frac{4}{20}$
X^2 P(X) :	$\frac{10}{20}$	$\frac{24}{20}$	$\frac{27}{20}$	$\frac{16}{20}$

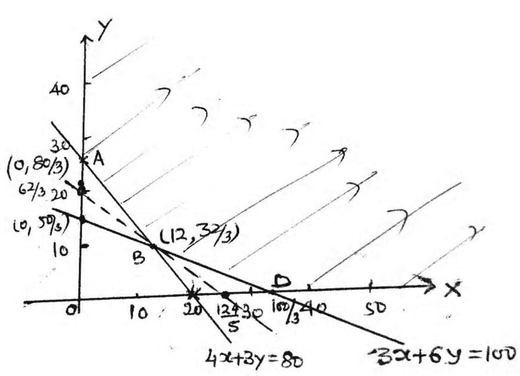
2

$1\frac{1}{2}$

Mean = $\sum X\,P(X) = \frac{35}{20} = \frac{7}{4}$ 1

Variance = $\sum X^2\,P(X) - \left[\sum X\,P(X)\right]^2 = \frac{77}{20} - \frac{49}{16} = \frac{63}{80}$ 1

21.



Let x units of F_1 and y units of F_2 be mixed

$\therefore$ We have Minimise cost (C) = $5x + 6y$ 1

subject to
$$\left.\begin{aligned} 4x + 3y &\geq 80 \\ 3x + 6y &\geq 100 \\ x \geq 0,\ y &\geq 0 \end{aligned}\right\}$$ 2

Correct Figure $1\frac{1}{2}$

C(A) = 160

C(B) = 60 + 64 = 124

C(D) = $\frac{500}{3}$

5x + 6y ≤ 124 passes through B only $\frac{1}{2}$

$\therefore$ Minimum cost = ₹ 124

F_1 = 12 units

$F_2 = \frac{32}{3}$ units 1

22. Let $\Delta = \begin{vmatrix} (b+c)^2 & a^2 & bc \\ (c+a)^2 & b^2 & ca \\ (a+b)^2 & c^2 & ab \end{vmatrix}$

$C_1 \to C_1 + C_2 - 2C_3 \Rightarrow \Delta = (a^2+b^2+c^2)\begin{vmatrix} 1 & a^2 & bc \\ 1 & b^2 & ca \\ 1 & c^2 & ab \end{vmatrix}$ 1

$R_1 \to R_1 - R_2$, and $R_2 \to R_2 - R_3 \Rightarrow \Delta = (a^2+b^2+c^2)\begin{vmatrix} 0 & a^2-b^2 & c(b-a) \\ 0 & b^2-c^2 & a(c-b) \\ 1 & c^2 & ab \end{vmatrix}$ $1\frac{1}{2}$

$= (a^2+b^2+c^2)(a-b)(b-c)\begin{vmatrix} 0 & a+b & -c \\ 0 & b+c & -a \\ 1 & c^2 & ab \end{vmatrix}$ $\frac{1}{2}$

$R_2 \to R_2 - R_1 \Rightarrow \Delta = (a^2+b^2+c^2)(a-b)(b-c)\begin{vmatrix} 0 & a+b & -c \\ 0 & c-a & c-a \\ 1 & c^2 & ab \end{vmatrix}$ 1

$\therefore \Delta = (a^2+b^2+c^2)(a-b)(b-c)(c-a)\begin{vmatrix} 0 & a+b & -c \\ 0 & 1 & 1 \\ 1 & c^2 & ab \end{vmatrix}$ 1

Expanding by C_1 to get $\Delta = (a^2+b^2+c^2)(a-b)(b-c)(c-a)(a+b+c)$ 1

OR

Let A = IA $\therefore \begin{pmatrix} 2 & -1 & 3 \\ -5 & 3 & 1 \\ -3 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$ 1

$R_2 \to R_2 - 2R_3 \Rightarrow \begin{pmatrix} 2 & -1 & 3 \\ 1 & -1 & -5 \\ -3 & 2 & 3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix} A$ $\frac{1}{2}$

$\begin{matrix} R_1 \to R_1 - 2R_2 \\ R_3 \to R_3 + 3R_2 \end{matrix} \Rightarrow \begin{pmatrix} 0 & 1 & 13 \\ 1 & -1 & -5 \\ 0 & -1 & -12 \end{pmatrix} = \begin{pmatrix} 1 & -2 & 4 \\ 0 & 1 & -2 \\ 0 & 3 & -5 \end{pmatrix} A$ 1

$R_1 \leftrightarrow R_2 \Rightarrow \begin{pmatrix} 1 & -1 & -5 \\ 0 & 1 & 13 \\ 0 & -1 & -12 \end{pmatrix} = \begin{pmatrix} 0 & 1 & -2 \\ 1 & -2 & 4 \\ 0 & 3 & -5 \end{pmatrix} A$ $\frac{1}{2}$

$\left\{\begin{matrix} R_1 \to R_1 + R_2 \\ R_3 \to R_3 + R_2 \end{matrix}\right. \Rightarrow \begin{pmatrix} 1 & 0 & 8 \\ 0 & 1 & 13 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 2 \\ 1 & -2 & 4 \\ 1 & 1 & -1 \end{pmatrix} A$ 1

$\begin{cases} R_1 \to R_1 - 8R_3 \\ R_2 \to R_2 - 13R_3 \end{cases} \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -7 & -9 & 10 \\ -12 & -15 & 17 \\ 1 & 1 & -1 \end{pmatrix} A$ 1

$\Rightarrow A^{-1} = \begin{pmatrix} -7 & -9 & 10 \\ -12 & -15 & 17 \\ 1 & 1 & -1 \end{pmatrix}$ 1

23. Points on the lines are $a_1 = (1, -1, 0)$, $a_2 = (0, 2, -1)$

and the direction of lines is $2\hat{i} - \hat{j} + 3\hat{k}$

let the equation of plane through a_1 be

$a(x-1) + b(y+1) + c(z) = 0$ ...(i) $\frac{1}{2}$

(0, 2, –1) lies on it, $\therefore -a + 3b - c = 0$ ...(ii) 1

and a, b, c are DR's of a line $\perp$ to the line with DR's 2, –1, 3

$\therefore 2a - b + 3c = 0$ ...(iii) 1

Solving (ii) & (iii) we get $\frac{a}{8} = \frac{b}{1} = \frac{c}{-5}$ 1

$\therefore$ Equation of plane is $8(x - 1) + 1(y + 1) - 5z = 0$

$\Rightarrow 8x + y - 5z = 7$ ...(iv) $\frac{1}{2}$

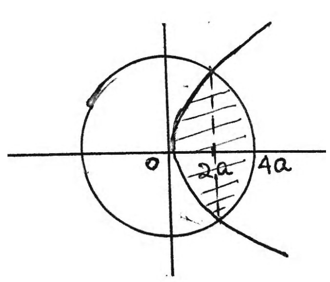
For the line $\frac{x-2}{3} = \frac{y-1}{1} = \frac{z-2}{5}$, since the point (2, 1, 2) lies on plane (iv)

as $8(2) + 1 - 5(2) = 7$ 1

and $3(8) + 1(1) + 5(-5) = 25 - 25 = 0$

$\therefore$ The plane (iv) contains the given line 1

24. Solving $y^2 = 6ax$ and $x^2 + y^2 = 16a^2$



we get $x^2 + 6ax - 16a^2 = 0$

$(x + 8a)(x - 2a) = 0$

$x = -8a$, $x = 2a$ 1

Correct Figure 1

Required area $= 2\left[\int_0^{2a} \sqrt{6a}\sqrt{x}\, dx + \int_{2a}^{4a} \sqrt{16a^2 - x^2}\, dx\right]$ 2

$= 2\left[\left(\sqrt{6}\sqrt{a}\,\frac{2}{3}x^{3/2}\right)_0^{2a} + \left(\frac{x}{2}\sqrt{16a^2 - x^2} + 8a^2 \sin^{-1}\frac{x}{4a}\right)_{2a}^{4a}\right]$ 1

$= 2\left[\frac{8\sqrt{3}a^2}{3} + 8a^2\frac{\pi}{2} - 2a^2\sqrt{3} - 8a^2\frac{\pi}{6}\right]$

$= 2\left[\frac{2\sqrt{3}a^2}{3} + 8a^2\frac{\pi}{3}\right]$ sq. units 1

25. Let $x_1, x_2 \in$ N and $f(x_1) = f(x_2)$

$\Rightarrow 4x_1^2 + 12x_1 + 15 = 4x_2^2 + 12x_2 + 15$

$\Rightarrow 4(x_1^2 - x_2^2) + 12(x_1 - x_2) = 0 \Rightarrow (x_1 - x_2)(4x_1 + 4x_2 + 12) = 0$

$\Rightarrow x_1 - x_2 = 0$ or $x_1 = x_2$ as $4x_1 + 4x_2 + 12 \neq 0$, $x_1, x_2 \in$ N

$\therefore f$ is a 1 – 1 function 2

f : N $\rightarrow$ S is onto as co-domain = range 1

Hence f is invertible.

$$y = 4x^2 + 12x + 15 = (2x+3)^2 + 6 \Rightarrow x = \frac{\sqrt{y-6}-3}{2}$$

$\therefore$ $f^{-1}(y) = \frac{\sqrt{y-6}-3}{2}$, $y \in$ S. 2

$f^{-1}(31) = \frac{\sqrt{31-6}-3}{2} = 1$ $\frac{1}{2}$

$f^{-1}(87) = \frac{\sqrt{87-6}-3}{2} = 3$ $\frac{1}{2}$

26. $f'(x) = 4x^3 - 24x^2 + 44x - 24$ 1

$= 4(x^3 - 6x^2 + 11x - 6) = 4(x-1)(x-2)(x-3)$ $1\frac{1}{2}$

$f'(x) = 0 \Rightarrow x = 1, x = 2, x = 3$ $\frac{1}{2}$

The intervals are $(-\infty, 1), (1, 2), (2, 3), (3, \infty)$ 1

since $f'(x) > 0$ in $(1, 2)$ and $(3, \infty)$

$\therefore$ $f(x)$ is strictly increasing in $(1, 2) \cup (3, \infty)$ 1

and strictly decreasing in $(-\infty, 1) \cup (2, 3)$ 1

OR

$f(x) = \sec x + 2 \log |\cos x|$

$f'(x) = \sec x \tan x - 2 \tan x = \tan x (\sec x - 2)$ 1

$f'(x) = 0 \Rightarrow \tan x = 0$ or $\sec x = 2$

$\Rightarrow x = \pi, x = \frac{\pi}{3}, \frac{5\pi}{3}$ $1\frac{1}{2}$

$f''(x) = \sec x \tan^2 x + (\sec x - 2) \sec^2 x$ 1

$$\left.\begin{array}{l} f''(\pi/3) = 6 \text{ (+ve)} \Rightarrow f(x) \text{ is minimum at } x = \pi/3 \\ f''(\pi) = -3 \text{ (–ve)} \Rightarrow f(x) \text{ is maximum at } x = \pi \\ f''(5\pi/3) = 6 \text{ (+ve)} \Rightarrow f(x) \text{ is minimum at } x = 5\pi/3 \end{array}\right\} \quad 1\frac{1}{2}$$

Maximum value $= f(\pi) = -1$. $\frac{1}{2}$

Minimum value $= f(\pi/3) = f(5\pi/3) = 2 - 2 \log 2$ or $2 + \log (1/4)$ $\frac{1}{2}$