JEE Mains 2018 Code C - Answers & Solutions

Q No.	Answer	Level	Chapter
1.	A	Medium	Ionic Equilibrium
2.	A	Medium	Introduction to Organic Chemistry
3.	C	Easy	P block - I
4.	D	Medium	Alcohols, Ethers and Phenols
5.	B	Hard	Redox Reactions
6.	A	Medium	Ionic Equilibrium
7.	C	Medium	Thermodynamics
8.	C	Medium	P - Block Elements - II
9.	B	Medium	Electrochemistry
10.	B	Easy	Chemical Bonding
11.	B	Easy	P block - I
12.	C	Easy	Chemical Bonding
13.	B	Hard	Amines
14.	B	Easy	Solid State
15.	B	Medium	Chemical Bonding
16.	C	Hard	Coordination Compounds
17.	B	Medium	Hydrogen
18.	D	Easy	Biomolecules, Polymers and Chemistry in Everyday life
19.	C	Hard	Amines
20.	B	Medium	Environmental Chemistry
21.	A	Medium	Coordination Compounds
22.	B	Easy	Hydrocarbons
23.	C	Hard	Basic Concepts of Chemistry
24.	B	Medium	Carboxylic Acids and Derivatives
25.	A	Medium	Hydrocarbons
26.	D	Medium	Chemical Equilibrium
27.	C	Medium	Alcohols, Ethers and Phenols
28.	B	Hard	Ionic Equilibrium
29.	D	Medium	Chemical Kinetics
30.	C	Medium	Solutions
31.	A	Medium	Indefinite Integration
32.	D	Medium	Conic Sections - II

33.	A	Medium	Conic Sections - II
34.	D	Medium	Vector Algebra
35.	B	Medium	Complex Numbers
36.	D	Hard	Definite Integration
37.	C	Hard	Binomial Theorem
38.	B	Medium	Sequence and Series
39.	B	Easy	Statistics
40.	D	Easy	Trigonometry
41.	A	Hard	Sets, Relations and Functions
42.	D	Medium	Permutations and Combinations
43.	C	Hard	Applications of Derivatives
44.	B	Hard	Limits
45.	C	Medium	Definite Integration
46.	A	Medium	Probability
47.	C	Medium	3 Dimensional Geometry
48.	A	Hard	Trigonometry
49.	B	Medium	Straight Lines
50.	A	Hard	Sequence and Series
51.	C	Medium	Conic Sections - II
52.	B	Medium	Conic Sections - I
53.	D	Hard	Continuity and Differentiability
54.	B	Medium	Matrices and Determinants
55.	D	Medium	Mathematical Reasoning
56.	A	Medium	Matrices and Determinants
57.	B	Medium	Inequalities
58.	C	Medium	Conic Sections - II
59.	B	Medium	Differential Equations
60.	A	Medium	3 Dimensional Geometry
61.	D	Hard	Wave Optics
62.	D	Hard	Modern Physics
63.	B	Easy	Semiconductor Electronics
64.	B	Easy	Units and Measurement
65.	A	Medium	Magnetic Effects of Electric Current
66.	A	Medium	Electrostatics
67.	A	Easy	Dynamics of Motion
68.	B	Medium	Work, Power, Energy and Momentum

69.	D	Medium	Capacitors
70.	A	Medium	Oscillations
71.	D	Medium	Modern Physics
72.	B	Medium	Magnetic Effects of Electric Current
73.	A	Easy	Current Electricity
74.	B	Easy	Electromagnetic Waves and Communication Systems
75.	B	Easy	Wave Optics
76.	B	Medium	Current Electricity
77.	D	Hard	Rotation
78.	A	Hard	Work, Power, Energy and Momentum
79.	B	Medium	Electromagnetic Waves and Communication Systems
80.	D	Easy	AC Circuits
81.	A	Medium	Motion in 1D
82.	A	Easy	Current Electricity
83.	B	Hard	Gravitation
84.	C	Medium	Modern Physics
85.	A	Easy	AC Circuits
86.	B	Medium	Thermodynamics and Kinetic Theory of Gases
87.	B	Medium	Mechanical Properties of Solids
88.	D	Hard	Waves
89.	D	Medium	Work, Power, Energy and Momentum
90.	C	Hard	Rotation

Solutions

Q1. (A)

CH_3COOK is a salt of weak acid CH_3COOH and strong base (KOH)

$\therefore$ The net solution will be basic.

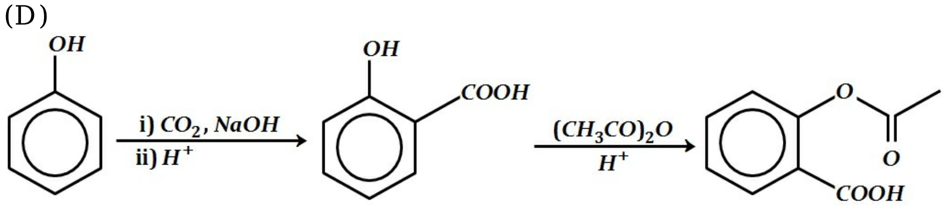
Q2. (A)

Kjeldahl's method is not applicable to compounds containing nitrogen in nitro $(-NO_2)$, azo groups $(-N_2^+)$ and nitrogen present in rings as nitrogen of these compounds does not convert to ammonium sulphate under the conditions of this method.

Q3. (C)

Both BCl_3 and $AlCl_3$ are lewis acids.

Q4. (D)



Q5. (B)

Weak base (WB) + Strong Acid (SA) $\rightarrow$ Acidic solution

When weak base is titrated against strong acid in the presence of methyl orange as indicator then the end point is obtained in acidic conditions $(pH < 7)$.

$\therefore$ The colour change of methyl orange is from yellow to pinkish red.

Q6. (A)

$$\begin{array}{ccccc} H_2S & \rightleftharpoons & 2H^+ & + & S^{2-} \\ 0.1 & & & & \\ 0.1-0.1\alpha & & 0.2\alpha+0.2 & & 0.1\alpha \end{array}$$

$K = K_1K_2 = 1\times10^{-7}\times1.2\times10^{-13} = 1.2\times10^{-20}$

$K = \frac{[H^+][S^{2-}]}{[H_2S]} \Rightarrow 1.2\times10^{-20} = \frac{0.2^2(1+\alpha)^2[S^{2-}]}{0.1(1-\alpha)}$

$\Rightarrow 1.2\times10^{-20} = \frac{0.2\times0.2\times[S^{2-}]}{0.1}$ $\left\{\begin{array}{c} 1+\alpha\approx1 \\ 1-\alpha\approx1 \\ \because \alpha \text{ is very small} \end{array}\right\}$

$\Rightarrow [S^{2-}] = 3\times10^{-20}$

Q7. (C)

$C_6H_6(l)+\frac{15}{2}O_2(g)\rightarrow 6CO_2(g)+3H_2O(l)$

Given $Q_v = -3263.9\ kJ/mole$

$\Rightarrow$ At constant volume, $\Delta U = Q_v$ (from 1st law)

$\Rightarrow \Delta U = -3263.9\ kJ/mole$

At constant pressure, $Q_p = ?$

$\Rightarrow$ From 1st law

$\Delta U = Q_p + W$

$\Rightarrow -3263.9 = Q_p - \Delta n_g RT$

$\Rightarrow -3263.9 = Q_p - \frac{(-1.5)\times8.314\times298}{1000}$

$\Rightarrow Q_P = -3263.9-3.7$

$= -3267.6\ kJ/mole$

Q8. (C)

$(NH_4)_2Cr_2O_7 \rightarrow Cr_2O_3 + N_2 + 4H_2O$

$NH_4NO_2 \rightarrow N_2 + 2H_2O$

$Ba(N_3)_2 \rightarrow Ba + 3N_2$

Q9. (B)

$$\begin{array}{ccccccc} B_2H_6 & + & 3O_2 & \rightleftharpoons & B_2O_3 & + & 3H_2O \\ 27.66\ g & & & & & & \\ =1\ mol & & 3\ mol & & 1\ mol & & 3\ mol \end{array}$$

$2H_2O \rightarrow 2H_2 + O_2$

Electrolysis of H_2O involves $4\ e^-$ transfer of $1\ mol\ O_2$

$\therefore$ For $3\ mol\ O_2$, $4\times3\ e^-$ transfer is required.

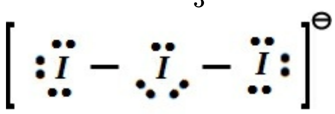
$\Rightarrow 12\ F$ of electricity $= 12\times96500\ C$

$\therefore 12\times96500 = It = 100\times t$

$\therefore t = 12\times965 = \frac{12\times965}{3600}hr = 3.2\ hr$

Q10. (B)

Structure of I_3^- is



Total number of lone pairs $= 9$

Q11. (B)

$$\begin{array}{ccccc} M & + & NaOH & \rightarrow & X \\ \downarrow & & & & \downarrow \\ Al & & & & Al(OH)_3 \\ & & & & \text{white gelatinous} \\ & & & & \text{precipitate} \end{array}$$

$X + NaOH \rightarrow$ dissolve

$$Al(OH)_3 + NaOH \rightarrow NaAlO_2$$

or

$$Na[Al(OH)_4]$$

Sodium aluminate

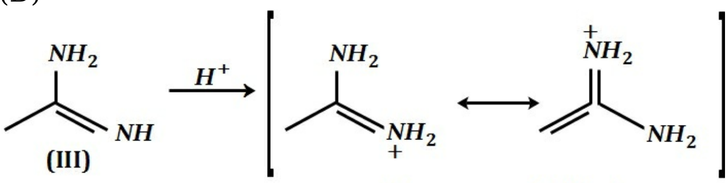
$X \xrightarrow{\Delta}$ oxide

$Al(OH)_3 \xrightarrow{\Delta} Al_2O_3$ (used in chromatography as adsorbent)

Q12. (C)

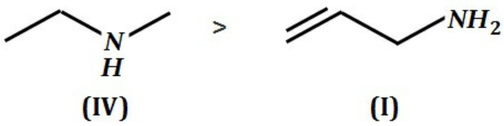
Electronic Configuration	Bond Order
$He_2^+ : \sigma 1s^2 \sigma^* 1s^1$	$+1/2$
$H_2^- : \sigma 1s^2 \sigma^* 1s^1$	$+1/2$
$H_2^{2-} : \sigma 1s^2 \sigma^* 1s^2$	0
$He_2^{2+} : \sigma 1s^2$	$+1$

Q13. (B)

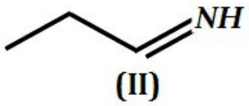


Resonance stabilized

The above compound forms resonance stabilized cation with H^+ and therefore is most acidic.
(IV) is secondary amine and (I) is primary amine. Secondary amines are more basic than primary amines.



In (II), the nitrogen is sp^2 hybridized unlike other alkyl amine which are sp^3 hybridized. Hence, it is the least basic compound.

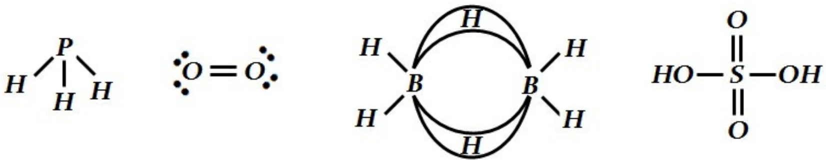


Q14. (B)

In frenkel defect, the cation is dislocated from its normal site to an interstitial site.

Q15. (B)

KCl is an ionic compound. PH_3, O_2, B_2H_6 and H_2SO_4 have covalent bonds.



Q16. (C)

$[Cr(H_2O)_6]Cl_3 \rightarrow$ Oxidation state $= +3$

$[Cr(C_6H_6)_2] \rightarrow$ Oxidation state $= 0$

$$K_2[Cr(CN)_2(O)_2(O_2)(NH_3)] \longrightarrow \underbrace{x+(-2)}_{2CN^{\ominus}}+\underbrace{(-2)}_{2O^{2-}}+\underbrace{0}_{1O_2}+\underbrace{0}_{1NH_3}=-2$$

$$\Rightarrow x = +4$$

Q17. (B)

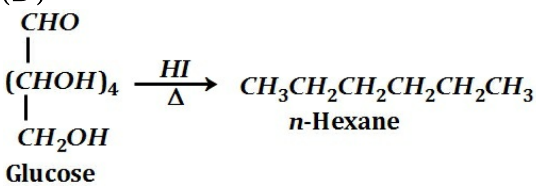
Oxidising action in acidic medium

$2Fe^{2+}(aq.)+2H^{+}(aq.)+H_2O_2(aq.)\rightarrow 2Fe^{3+}(aq.)+2H_2O(l)$

Reducing action in basic medium

$2Fe^{3+}(aq.)+2OH^{-}+H_2O_2(aq.)\rightarrow 2Fe^{2+}(aq.)+2H_2O+O_2$

Q18. (D)



Q19. (C)

This is a wrong question in JEE 2018. The question asks about histamine. However, the pK_a given is of histidine. Also, pH of human blood is basic and this information is missing.

Solution:

pH of human blood is 7.35

pK_a of the two nitrogens in aromatic ring $=5.8$

pK_a of aliphatic nitrogen $=9.4$

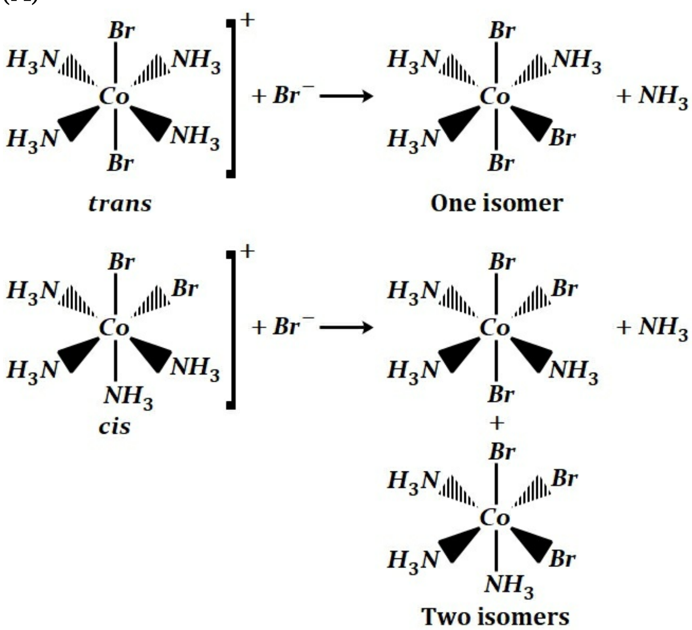
The two nitrogens in aromatic ring are equivalent as they show tautomerism.

At pH 7.35 (human blood), aliphatic nitrogen can get protonated but not aromatic nitrogen.

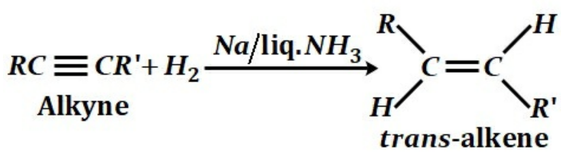
Q20. (B)

$[3Ca_3(PO_4)_2\cdot Ca(OH)_2]+F^{-}\rightarrow[3Ca_3(PO_4)_2\cdot CaF_2]$

Q21. (A)



Q22. (B)



Q23. (C)

$$C_xH_y + \frac{\left(2x+\frac{y}{2}\right)}{2}O_2 \to x\,CO_2 + \frac{y}{2}\,H_2O$$

$\Rightarrow\ x+\frac{y}{4}$ is the complete amount of oxygen (O_2) required to burn C_xHy.

$\because$ Since $C_xH_yO_z$ contains half as much as oxygen required.

$\Rightarrow 2z$ is the complete amount of oxygen required.

$\Rightarrow 2z = \left(x+\frac{y}{4}\right)\times 2 \left\{\begin{array}{l}\because \text{Counting}\\ \text{atoms of oxygen}\end{array}\right.$

$\Rightarrow z = x+\frac{y}{4}$

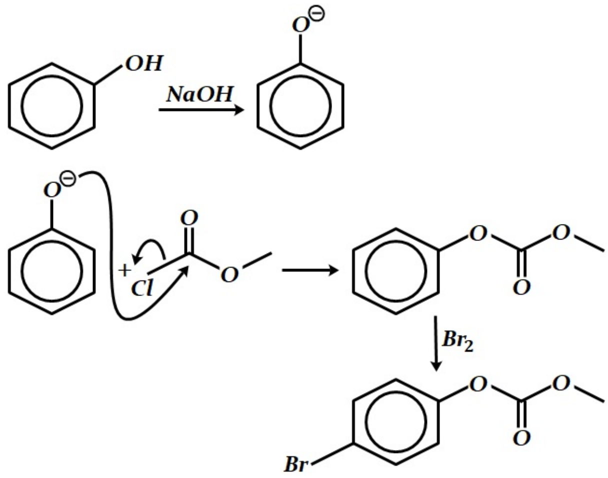
for option C, $C_2H_4O_3$

$\Rightarrow x=2;\ y=4;\ z=3$

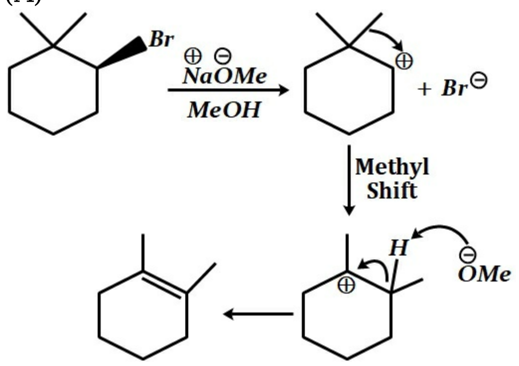
$\Rightarrow z = x+\frac{y}{4} = 2+\frac{4}{4} = 2+1 = 3$

Hence, Option C is correct.

Q24. (B)



Q25. (A)



Q26. (D)

For exothermic reaction, $\Delta H = -ve$

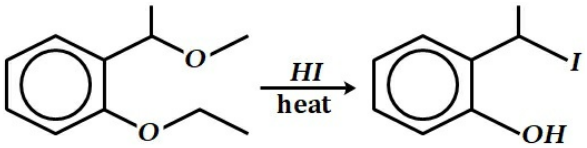
$\therefore \Delta G = \Delta H - T\Delta S = -ve$

$\Delta G = -RT\ln K = -ve$

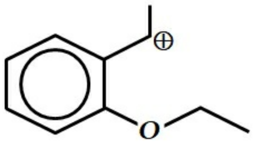
$\Rightarrow \ln K \propto \frac{1}{T}$

Only lines A and B shows that $\ln K$ increases with $\frac{1}{T}$

Q27. (C)



Intermediate is stable carbocation



Q28. (B)

$BaSO_4 \rightleftharpoons Ba^{2+} + SO_4^{2-}$
$K_{sp} = [Ba^{2+}] \cdot [SO_4^{2-}]$
Given, $Na_2SO_4 = 1\ M,\ V_{Na_2SO_4} = 50\ mL$
$\Rightarrow$ mmoles of $Na_2SO_4 = 50 \times 1 = 50$ mmoles
$\Rightarrow$ mmoles of $SO_4^{2-} = 50$ mmoles.
Final volume of solution $= 500\ mL$
$\therefore$ concentration of $[SO_4^{2-}] = \frac{50}{500} = 0.1\ M$
$\therefore K_{sp} = [Ba^{2+}][SO_4^{2-}]$
$\Rightarrow 1 \times 10^{-10} = [Ba^{2+}][0.1]$
$\Rightarrow [Ba^{2+}] = 1 \times 10^{-9}\ M$
$\Rightarrow$ The conc. of $[Ba^{2+}]$ in the final $500\ mL$ solution is $1 \times 10^{-9}\ M$
$\therefore$ mmoles of $[Ba^{2+}] = 1 \times 10^{-9} \times 500$ mmoles
$\because$ final volume $= 500\ mL$
& Vol of Na_2SO_4 added $= 50\ mL$
$\therefore$ Initial volume $= 500 - 50 = 450\ mL$
$\therefore$ concentration of Ba^{2+} initially $= \frac{1 \times 10^{-9} \times 500}{450} = 1.1 \times 10^{-9}\ M$

Q29. (D)

Let rate of decomposition of acetaldehyde be $R = K\ [CH_3CHO]^x$
when 5% reacted $R = 1\ Torr/s$
$\Rightarrow R = K\ [P_o(1-0.05)]^x$
$1 = K[P_o \times 0.95]^x$ (1)
when 33% reacted, $R = 0.5\ Torr/s$
$\Rightarrow R = K\ [P_o(1-0.33)]^x$
$0.5 = K\ [P_o \times 0.67]^x$ (2)
$\Rightarrow$ Taking ratio of equations $1\ \&\ 2$ we get
$\frac{1}{0.5} = \left(\frac{P_o \times 0.95}{P_o \times 0.67}\right)^x$
$\Rightarrow 2 = (1.414)^x = (\sqrt{2})^x$
$\Rightarrow 2 = (2)^{x/2}$
$\Rightarrow \frac{x}{2} = 1$
$\Rightarrow x = 2$
$\therefore$ The order of the reaction is 2.

Q30. (C)

$\Delta T_f = i\ k_f \cdot m$
For 1 modal aqueous solution, the compound with lowest value of i will have highest freezing point.
$[Co(H_2O)_5Cl]Cl_2 \cdot H_2O \Rightarrow i = 3$
$[Co(H_2O)_4Cl_2]Cl \cdot 2H_2O \Rightarrow i = 2$
$[Co(H_2O)_3 \cdot Cl_3] \cdot 3H_2O \Rightarrow i = 1$
$[Co(H_2O)_6]Cl_3 \Rightarrow i = 4$
$\therefore$ Option C will have the highest freezing point.

Q31. (A)

$\int \frac{\sin^2 x \cos^2 x\ dx}{(\sin^5 x+\cos^3 x+\sin^3 x\cos^2 x+\cos^5 x)^2}$

$\int \frac{\sin^2 x \cos^2 x\ dx}{(\sin^2 x+\cos^2 x)^2(\sin^2 x+\cos^3 x)^2}$

$\int \frac{\sin^2 x \cos^2 x\ dx}{(\sin^3 x+\cos^3 x)^2}$

Divide by $\cos^6 x$

$\int \frac{\frac{\sin^2 x}{\cos^2 x}\frac{1}{\cos^2 x}dx}{(\tan^3 x+1)^2}$

$\int \frac{\tan^2 x \sec^2 x\ dx}{(1+\tan^3 x)^2}$

$\Rightarrow 1+\tan^3 x = t$

$3\tan^2 x \sec^2 x\ dx = dt$

$\Rightarrow \frac{1}{3}\frac{t^{-2}+1}{-2+1}+C$

$\Rightarrow \frac{1}{3}\left(\frac{-1}{t}\right)+C$

$\Rightarrow \frac{-1}{3(1+\tan^3 x)}+C$

Q32. (D)

$4x^2-y^2=36$

Tangents at P & Q intersect at $T(0,3)$

Chord of contact $= PQ$

$4xx_1-yy_1=36$

$4(0)x_1-y(3)=36$

$-3y=36$

$y=-12$

$y=-12 \qquad x=\pm 3\sqrt{5}$

$P(3\sqrt{5},-12) \quad Q(-3\sqrt{5},12) \qquad T(0,3)$

Area $= \left|\frac{1}{2}\begin{vmatrix} 3\sqrt{5} & -12 & 1 \\ -3\sqrt{5} & -12 & 1 \\ 0 & 3 & 1 \end{vmatrix}\right|$

$= \left|\frac{1}{2}\times 90\sqrt{5}\right| = 45\sqrt{5}$

Q33. (A)

$y^2 = 16x \quad P(16,16)$
$\angle CPB = \theta \quad \tan\theta = ?$
$yy_1 = 8(x+x_1)$
$16y = 8(x+16)$
$2y = x+16$ (equation of tangent)
$MT = \frac{1}{2}$
$MN = -2$
$(y-16) = -2(x-16)$
$y-16 = -2x+32$
$2x+y-48=0$ (equation of normal)

$y=0 \quad x=-16$
$A(-16,0) \quad P(16,16)$
$y=0 \quad x=24$
$B \equiv (+24,0)$
$C \equiv (4,0)$
$\theta =$ angle between PC & PB
$m_{PC} = \frac{16-0}{12} = \frac{4}{3}$
$m_{PB} = \frac{16-0}{-8} = -2$
$\tan\theta = \left|\frac{m_1-m_2}{1+m_1m_2}\right| = \left|\frac{\frac{4}{3}+2}{1+\left(\frac{4}{3}\right)(-2)}\right|$
$= \left|\frac{\frac{10}{3}}{\frac{-5}{3}}\right| = 2$

Q34. (D)

$\vec{u} = x(2\hat{i}+3\hat{j}-\hat{k})+y(\hat{j}+\hat{k})$
$\vec{u} = \hat{i}(2x)+\hat{j}(3x+y)+\hat{k}(-x+y)$
$\vec{u}-\vec{b} = 24 \quad \vec{u}\cdot\vec{a} = 0$
$(2x\hat{i}+(3x+y)\hat{j}+(-x+y)\hat{k})\cdot(\hat{j}+\hat{k}) = 24$
$3x+y+(-x+y) = 24$
$2x+2y = 24$
$x+y = -12$ ...(i)
$(2x\hat{i}+(3x+y)\hat{j}+(-x+y)\hat{k})\cdot(2\hat{i}+3\hat{j}-\hat{k}) = 0$
$4x+3(3x+y)-1(-x+y) = 0$
$4x+9x+3y+x-y = 0$
$14x+2y = 0$
$4x+y = 0$
$y = -7x$ ...(ii)
Solving (i) & (ii) , we get
$-6x = 12$
$x = -2$
$y = 14$
$\vec{u} = 2(-2)\hat{i}+(-6+14)\hat{j}+(+2+14)\hat{k}$
$= -4\hat{i}+8\hat{j}+16\hat{k}$
$|\vec{u}|^2 = (\sqrt{16+64+256})^2$
$= 336$

Q35. (B)

$\alpha, \beta \in C$

$x^2 - x + 1 = 0 \qquad \alpha^{101} + \beta^{107} = ?$

$D = (-1)^2 - 4(1)(1)$

$= 1 - 4 = -3$

$\alpha, \beta = \frac{+1 \pm \sqrt{3}i}{2}$

$\alpha = \frac{1}{2} + \frac{\sqrt{3}i}{2} \qquad \beta = \frac{1}{2} - \frac{\sqrt{3}i}{2}$

$\alpha^2 = -\beta$

$\alpha^3 = \beta^3 = -1$

$\Rightarrow \alpha^{99} \cdot \alpha^2 + \beta^{105} \cdot \beta^2$

$\Rightarrow \alpha^{99} \cdot \alpha^2 + \beta^{105} \cdot \beta^2$

$\Rightarrow -\alpha^2 - \beta^2$

$\Rightarrow -(\alpha^2 + \beta^2)$

$\Rightarrow -(\beta^2 - \beta) \cdot (\beta^2 - \beta) = -1$

$\Rightarrow -1(-1) = 1$

Q36. (D)

$g(x) = \cos x^2 \qquad f(x) = \sqrt{x}$

$\alpha < \beta \qquad 18x^2 - 9\pi x + \pi^2 = 0$

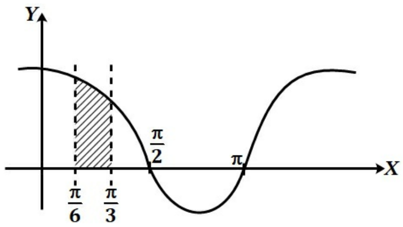
$18x^2 - 6\pi x - 3\pi x + x^2 = 0$

$6x(3x - \pi) - \pi(3x - \pi) = 0$

$x = \frac{\pi}{6}, \frac{\pi}{3}$

$\alpha = \frac{\pi}{6}, \beta = \frac{\pi}{3}$

$y = (gof)(x) \qquad y = \cos|x| = \cos x$



Shaded region,

$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \cos x \, dx$

$= (\sin x)_{\frac{\pi}{6}}^{\frac{\pi}{3}}$

$= \frac{\sqrt{3}}{2} - \frac{1}{2}$

Q37. (C)

$\left(x + \sqrt{x^3 - 1}\right)^5 + \left(x - \sqrt{x^3 - 1}\right)^5 \qquad (x > 1)$

Sum of coefficient of odd degrees

$a = x \quad b = \sqrt{x^3 - 1}$

$\Rightarrow (a + b)^5 + (a - b)^5$

$\Rightarrow 2\left(a^5 + 10a^3b^2 + 5ab^4\right)$

$\Rightarrow 2\left(x^5 + 10x^3\left(x^3 - 1\right) + 5x\left(x^3 - 1\right)^2\right)$

$\Rightarrow 2\left(x^5 + 10x^6 - 10^3 + 5x\left(x^6 + 1 - 2x^3\right)\right)$

$\Rightarrow 2x^5 + 20x^6 - 20x^3 + 10x^7 + 10x - 20x^4$

$\Rightarrow 10x^7 + 2x^5 - 20x^3 + 10x + 20x^6 - 20x^4$

Sum of coefficients $= 10 + 2 - 20 + 10$

$= 2$

Q38. (B)

Given, $\sum_{k=0}^{12} a_{4k+1} = 416$

$\Rightarrow a_1 + a_5 + a_9 + a_{13} + \ldots + a_{49} = 416$

$\Rightarrow a_1 + (a_1 + 4d) + (a_1 + 8d) + \ldots + (a_1 + 48d) = 416$

$\Rightarrow S_{13} = 416$

$\Rightarrow \frac{13}{2}[a_1 + (a_1 + 48d)] = 416$

$\Rightarrow 2a_1 + 48d = \frac{2 \times \cancel{416}^{32}}{\cancel{13}} = 64$

$\Rightarrow a_1 + 24Ad = 32$ ....(1)

Also, $a_9 + a_{43} = 66$

$\Rightarrow a_1 + 8d + a_1 + 42d = 66$

$\Rightarrow 2a_1 + 50d = 66$

$\Rightarrow a_1 + 25d = 33$ ....(2)

From eqn. (1) & (2), we get:

$a_1 = 8$ and $d = 1$

Now, $a_1^2 + a_2^2 + \ldots + a_{17}^2 = 140\, m$

$\sum_{i=1}^{17} a_1^2 = 140\, m$

$\Rightarrow \sum_{i=1}^{17} (a_1 + (i-1)\, d)^{\ 2} = 140\, m$

$\Rightarrow \sum_{i=1}^{17} [8 + (i-1)]^{\ 2} = 140\, m$

$\Rightarrow \sum_{i=1}^{17} (7+i) = 140\, m$

$\Rightarrow \sum_{i=1}^{17} 49 + i^2 + 14\, i = 140\, m$

$\Rightarrow 49 \sum_{i=1}^{17} + \sum_{i=1}^{17} i^2 + 14 \sum_{i=1}^{17} i = 140\, m$

$\Rightarrow 49\,(17) + \frac{i(i+1)(2i+1)}{6} + \frac{14 \times 17 \times 18}{2} = 140\, m$

$\Rightarrow 833 + \frac{17 \times \cancel{18}^3 \times 36}{\cancel{6}} + 2142 = 140\, m$

$\Rightarrow 4760 = 140\, m$

$\Rightarrow m = \frac{476\cancel{0}}{14\cancel{0}} = 34$

Q39. (B)

Given, $\sum_{i=1}^{9}(x_i - 5) = 9$

$\Rightarrow \sum_{i=1}^{9} -x_i - 5\sum_{i=1}^{9} 1 = 9$

$\Rightarrow \sum_{i=1}^{9} x_i = 9 + 5(9)$

$\Rightarrow \sum_{i=1}^{9} x_i = 54$ ....(1)

Also, $\sum_{i=1}^{9}(x_i - 5)^2 = 45$

$\Rightarrow \sum_{i=1}^{9} x_i^2 + 25 - 10x_i = 45$

$\Rightarrow \sum_{i=1}^{9} x_i^2 + 25\sum_{i=1}^{9} 1 - 10\sum_{i=1}^{9} x_i = 45$

$\Rightarrow \sum_{i=1}^{9} x_i^2 + 25 \times 9 - 10(54) = 45$

$\Rightarrow \sum_{i=1}^{9} x_i^2 = 45 + 540 - 225 = 360$ ....(2)

Now, $\sigma^2 = \frac{1}{n} \sum x_i^2 - \left(\frac{\sum x_i}{n}\right)^2$

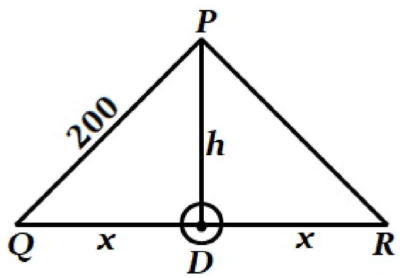
$= \frac{1}{9}\sum_{i=1}^{9} x_i^2 - \left(\frac{1}{9}\sum_{i=1}^{9} x_i\right)^2$

$= \frac{360}{9} - \left(\frac{54}{9}\right)^2$ [From eqn.(1) & (2)]

$= 40 - (6)^2 = 4$

$\therefore$ Standard deviation, $\sigma = \sqrt{4} = 2$

Q40. (D)



$\tan 30^\circ = \frac{h}{x}$ $\quad \tan 45^\circ = \frac{h}{PD}$

$\frac{1}{\sqrt{3}} = \frac{h}{x}$ $\qquad PD = h$

$\angle PDQ = 90^\circ$

$h^2 + x^2 = (200)^2$

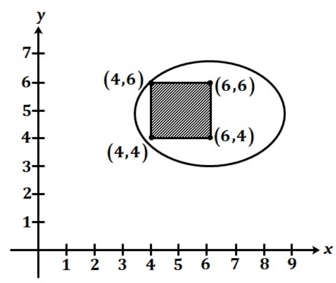
$h^2 + 3h^2 = (200)^2$

$4h^2 = 50 \times 200$

$h^2 = 50 \times 200 = 10000$

$h = 100\,m$.

Q41. (A)



We have,

$|a-5| < 1$

then, $-1 < a-5 < 1$

or $4 < a < 6$

And $|b-5| < 1$

then, $-1 < b-5 < 1$

$4 < a < 6$

Now, $A = \{(a, b)\}$

Also, $4(a-5)^2 + 9(b-5)^2 \leq 36$

$\frac{(a-5)^2}{9} + \frac{(b-5)^2}{4} \leq 1$

All the points in set A will lie inside the rectangle.

Therefore, $A \subset B$.

Q42. (D)

6 different Novels

$N_1, N_2, \ldots, N_6$

3 different dictionaries

D_1 D_2 D_3

4 Novels 1 Dictionary

$$\begin{array}{ccccc} N_1 & N_2 & D_1 & N_5 & N_4 \\ 6 & 5 & D_1 & 4 & 3 \\ & & 3 & & \end{array}$$

$6 \times 5 \times 4 \times 3 \times 3 = 30 \times 4 \times 9 = 36 \times 30$

$= 1080.$

Q43. (C)

$h\left(x\right) = \frac{x^2 + \frac{1}{x^2}}{x - \frac{1}{x}} = \left(x - \frac{1}{x}\right) + \frac{2}{x - \frac{1}{x}}$

when $x - \frac{1}{x} < 0 \Rightarrow x - \frac{1}{x} + \frac{2}{x - \frac{1}{x}} \leq -2\sqrt{2}$

so $-2\sqrt{2}$ will be local maximum value

when $x - \frac{1}{x} > 0 \Rightarrow x - \frac{1}{x} + \frac{2}{x - \frac{1}{x}} \geq 2\sqrt{2}$

so $2\sqrt{2}$ will be local minimum value.

Q44. (B)

$\lim\limits_{x \to 0^+} x\left(\left[\frac{1}{x}\right] + \left[\frac{2}{x}\right] + \ldots + \left[\frac{15}{x}\right]\right)$

$\lim\limits_{x \to 0^+} x\left(\frac{1+2+3+\ldots+15}{x}\right) - \left(\left\{\frac{1}{x}\right\} + \left\{\frac{2}{x}\right\} + \ldots + \left\{\frac{15}{x}\right\}\right)$

$\because 0 \leq \left\{\frac{r}{x}\right\} < 1$

$0 \leq x\left\{\frac{r}{x}\right\} < x$

$\lim\limits_{x \to 0^+} x\left(\frac{1+2+3+\ldots+15}{x}\right) = \frac{15.16}{2} = 120.$

Q45. (C)

$$\int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x} dx$$

$$I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^x} dx \quad (1)$$

$f(x) = f(a+b-x)$

$$I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x}{1+2^{-x}} dx$$

$$I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \left(\frac{\sin^2 x}{2^x+1}\right)^{2x} dx \quad (2)$$

Add (1) & (2)

$$2I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{\sin^2 x(2^x+1)}{(2^x+1)} dx$$

$$2I = \int_{\frac{-\pi}{2}}^{\frac{\pi}{2}} \frac{1-\cos 2x}{2} dx$$

$2I = \left(\frac{x}{2} - \frac{\sin 2x}{4}\right)_{\frac{-\pi}{2}}^{\frac{\pi}{2}}$

$2I = \left(\frac{\pi}{4} - 0\right) - \left(\frac{-\pi}{4} - 0\right)$

$2I = \frac{\pi}{4} \times 2 \Rightarrow I = \frac{\pi}{4}.$

Q46. (A)

Case I

Event 1: Taking out a red ball.

Event 2: Taking out a red ball after adding two red balls.

$\Rightarrow \frac{4}{10} \times \frac{6}{12} = \frac{1}{5}$

Case II

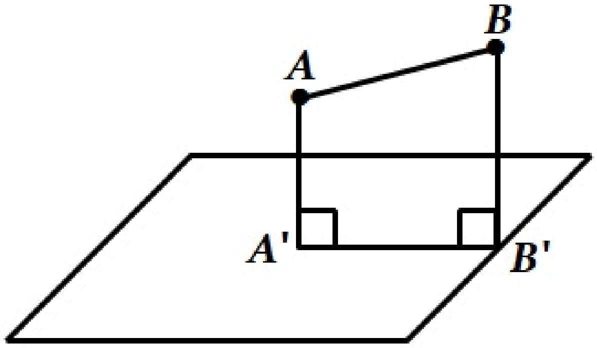
Event 1: Taking out a black ball.

Event 2: Taking out a red ball after adding two black balls.

$\Rightarrow \frac{6}{10} \times \frac{4}{12} = \frac{1}{5}$

Total probability $= \frac{1}{5} + \frac{1}{5} = \frac{2}{5}$.

Q47. (C)



Let $a(5,-1,4)$ and $B(4,-1,3)$

Plane: $x+y+z=7$

$\Rightarrow \vec{n} = \hat{i}+\hat{j}+\hat{k}$

$DR's = (1,1,1)$

Eqn. of AA'

$\Rightarrow \frac{x-5}{1} = \frac{y+1}{1} = \frac{z-4}{1}$

$= \lambda$

$\Rightarrow x = \lambda+5,\ y = \lambda$

$-1,\ z = \lambda+4$

$\Rightarrow A' \equiv (\lambda+5,\ \lambda-1,$

$\lambda+4)$

$\because A'$ lies on plane $\Rightarrow \lambda+5+\lambda-1+\lambda$

$+4 = 7$

$\Rightarrow 3\lambda+8=7$

$\Rightarrow \lambda = \frac{-1}{3}$

$\Rightarrow A' \equiv \left(\frac{14}{3}, \frac{-4}{3}, \frac{11}{3}\right)$

Similarly, Eqn. of $BB' = \frac{x-4}{1} = \frac{y+1}{1}$

$= \frac{z-3}{1}$

$\Rightarrow \frac{x-4}{1} = \frac{y+1}{1} = \frac{z-3}{1}$

$= \mu$

$\because B'$ lies on the plane $\Rightarrow \mu+4+\mu-1+\mu$

$+3 = 7$

$= 3\mu = +1 \Rightarrow \mu = +\frac{1}{3}$

$\Rightarrow B' \equiv \left(\frac{13}{3}, \frac{-2}{3}, \frac{10}{3}\right)$

$\Rightarrow A'B'$

$= \sqrt{\left(\frac{14}{3}, \frac{13}{3}\right)^2 + \left(\frac{-14}{3} + \frac{2}{3}\right)^2 + \left(\frac{11}{3} - \frac{10}{3}\right)^2}$

$\sqrt{\frac{1}{9} + \frac{4}{9} + \frac{1}{9}} = \sqrt{\frac{2}{3}}$

$= \sqrt{\frac{2}{3}}$

Q48. (A)

$8\ \cos x\left[\cos\ \left(\frac{\pi}{6}+x\right)\cos\left(\frac{\pi}{6}-x\right)'-\frac{1}{2}\right]=1$

$4\ \cos x\left[\cos\frac{\pi}{3}+\cos 2x-1\right]=1$

$4\ \cos x\left[\cos 2x-\frac{1}{2}\right]=1$

$4\ \cos x\left[2\cos^2 x-1-\frac{1}{2}\right]=1$

$2\ \cos x\left[4\ \cos^2 x-3\right]=1$

$4\ \cos^3 x-3\ \cos x=\frac{1}{2}$

$\cos 3x=\cos\frac{\pi}{3}$

$3x=2n\pi\pm\frac{\pi}{3}$

$n=0,\ \ 3x\pm\frac{\pi}{3},\quad x=\pm\frac{\pi}{9},\ \ x=\frac{\pi}{9}$

$n=1,\ 3x=2\pi\pm\frac{\pi}{3}$

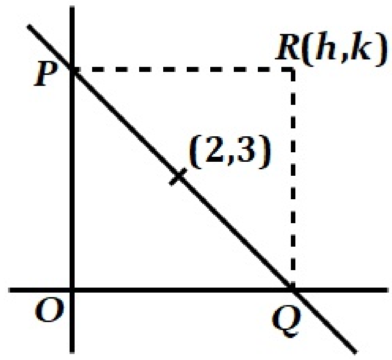
$3x=\frac{5\pi}{3},\frac{7\pi}{3},\ x=\frac{5\pi}{9},\ \frac{7\pi}{9}$

Rest all solutions are not in $[0,\ \pi]$

$\text{Sum}=\frac{\pi}{9}+\frac{5\pi}{9}+\frac{7\pi}{9}=\frac{13\pi}{9}$

So $k=\frac{13}{9}$

Q49. (B)



Let the slope of line PQ be m

$\Rightarrow$ Eqn. of line $PQ=y-3=m(x-2)$

$\Rightarrow mx-y+3-2m=0$

Coordinates of $P\equiv(0,3-2m)$

Coordinates of $Q\equiv\left(\frac{2m-3}{m},0\right)$

$\Rightarrow h=\frac{2m-3}{m}$

$\Rightarrow hm=2m-3$

$\Rightarrow(2m-hm)=3$

$\Rightarrow m=\frac{3}{2-h}$ ....(1)

& $k=3-2m$

$\Rightarrow m=\frac{3-k}{2}$ ....(2)

$\Rightarrow(1)=(2)$

$\Rightarrow\frac{3}{2-h}=\frac{3-k}{2}$

$\Rightarrow 6=6-2k-3h+hk$

$\Rightarrow 3h+2k=hk$

$\Rightarrow 3x+2y=xy$

Q50. (A)

$B-2A=\sum_{r=1}^{40}T_r-2\sum_{r=1}^{20}T_r$

$=\sum_{r=21}^{40}T_r-\sum_{r=1}^{20}T_r$

$B-2A=\left(21^2+2.22^2+23^2+2.24^2+\ldots+40^2\right)$

$-\left(1^2+2.2^2+3^2+2.4^2+\ldots+20^2\right)$

$=20\left[22+2.24+26+2.28+\ldots+60\right]$

$$=20\left[\underbrace{(22+24+26\ldots60}_{20\text{ terms}}+\underbrace{24+28+\ldots+60)}_{10\text{ terms}}\right]$$

$=20\left[\frac{20}{2}(22+60)+\frac{10}{2}(24+60)\right]$

$=10\left[20.82+10.84\right]$

$=100\left[164+84\right]=100\times 248$

Q51. (C)
(Set -C)
$\Rightarrow$ Let $POI = (x_1,\ y_1)$
Slope of tangent on ellipse $= \frac{-9x_1}{by_1}$
Slope of tangent on paprabola $= \frac{3}{y_1}$
$\Rightarrow$ curves are at right angle, hence
$\Rightarrow \frac{-9x_1}{by_1} \times \frac{3}{y_1} = -1$
$\Rightarrow by_1^2 = 27x_1$
$\Rightarrow b\left(6x_1\right) = 27x_1$
$\Rightarrow b = \frac{27}{6} = \frac{9}{2}$

Q52. (B)
Orthocentre $A\left(-3,\ 5\right)$ centroid $B(3,\ 3)$ and $AB = \sqrt{40} = 2\sqrt{10}$

Centroid divides orthocentre and circumcentre in ratio $2:1$
$\therefore AB : BC = 2:1$
Now $AB = \frac{2}{3}AC$
$AC = \frac{3}{2}AB = \frac{3}{2}\left(2\sqrt{10}\right)$
$AC = 3\sqrt{10}$
Radius of circle with AC as diametre is
$\frac{3}{2}\sqrt{10} = 3\sqrt{\frac{5}{2}}$.

Q53. (D)
$f\left(x\right) = |x-\pi|\left(e^{|x|}-1\right)\sin|x|$
we check differentiability at $x = \pi$ & $x = 0$
At $x = 0$
$R.H.D. = \lim\limits_{h\to 0^+} \frac{|\pi+h-\pi|\left(e^{|\pi+h|}-1\right)\sin|\pi+h|-0}{h} = 0$
$L.H.D. = \lim\limits_{h\to 0^+} \frac{|\pi-h-\pi|\left(e^{|\pi-h|}-1\right)\sin|\pi-h|-0}{-h} = 0$
$\because RHD = LHD$, so function is differentiable at $x = \pi$
At $x = \pi$
$R.H.D. = \lim\limits_{h\to 0^+} \frac{|h-\pi|\left(e^{|h|}-1\right)\sin|h|-0}{h} = 0$
$L.H.D = \lim\limits_{h\to 0^+} \frac{|-h-\pi|\left(e^{|-h|}-1\right)\sin|-h|-0}{-h} = 0$
$\because RHD = LHD$, so function is differentiable at $x = \pi$
$\therefore$ set S is empty set, ϕ.

Q54. (B)

$$\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (A+Bx)(x-A)^2$$

Put $x=0 \Rightarrow \begin{vmatrix} -4 & 0 & 0 \\ 0 & -4 & 0 \\ 0 & 0 & -4 \end{vmatrix} = A^3 \Rightarrow A=-4$

$$\begin{vmatrix} x-4 & 2x & 2x \\ 2x & x-4 & 2x \\ 2x & 2x & x-4 \end{vmatrix} = (Bx-4)(x+4)^2$$

$$\begin{vmatrix} 1-\frac{4}{x} & 2 & 2 \\ 2 & 1-\frac{4}{x} & 2 \\ 2 & 2 & 1-\frac{4}{x} \end{vmatrix} = \left(B-\frac{4}{x}\right)\left(1+\frac{4}{x}\right)^2$$

Put $x \to \infty \Rightarrow \begin{vmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{vmatrix} = B \Rightarrow B=5$

ordered pair $(A,\ B)$ is $(-4,\ 5)$.

Q55. (D)

p	q	$\sim p$	$\sim q$	$p\vee q$	$\sim(p\vee q)$	$\sim p\wedge q$	$(p\vee q)\vee(\sim p\wedge q)$
0	0	1	1	0	1	0	1
0	1	1	0	1	0	1	1
1	0	0	1	1	0	0	0
1	1	0	0	1	0	0	0

$\sim(p\vee q)\vee(\sim p\wedge q)$ is equivalent to $\sim p$.

Q56. (A)

$x+ky+3z=0$ (1)

$3x+ky-2z=0$ (2)

$2x+4y-3z=0$ (3)

Eq (2) − Eq (1)

$2x-5z=0$ (4)

from eq (3) & (4), we get

$5z+4y-3z=0 \Rightarrow 2z+4y=0$

$\Rightarrow \frac{z}{y}=-2$

also $\frac{5}{2}\cdot z\cdot\frac{z}{y^2}=\frac{5}{2}\frac{z^2}{y^2}=\frac{5}{2}(-2)^2=10$

Q57. (B)

$2|\sqrt{x}-3|+\sqrt{x}(\sqrt{x}-6)+6=0$

Case - I: $\sqrt{x} \geq 3$

$\Rightarrow 2\sqrt{x}-\cancel{6}+x-6\sqrt{x}+\cancel{6}=0$ (1)

$\Rightarrow x-4\sqrt{x}=0$

$\Rightarrow \sqrt{x}(\sqrt{x}-4)=0$

$\Rightarrow \sqrt{x}=0$ or $\sqrt{x}=4$

$\Rightarrow x=0$ or $x=16$

But $\sqrt{x} \geq 3 \Rightarrow x \geq 9$

Hence, $x=16$ is accepted &

$x=0$ is rejected

Case - II: $0 \leq \sqrt{x} < 3$

$\Rightarrow -2\sqrt{x}+6+x-6\sqrt{x}+6=0$

$\Rightarrow -8\sqrt{x}+x+12=0$

$\Rightarrow 64x+x^2+144+24x$

$\Rightarrow x^2-40x+144=0$

$\Rightarrow (x-36)(x-4)=0$

$\Rightarrow x=36, x=4$

$\Rightarrow \sqrt{x}=6$ or $\sqrt{x}=2$

$[\because x \geq 0]$

$\therefore \sqrt{x}=6$ is rejected because $\sqrt{x}<3$

$\Rightarrow \sqrt{x}=2 \Rightarrow x=4$

$\therefore S=\{4,16\}$

So S contains only 2 elements

Option (B).

Q58. (C)

$y=x^2+6$

$x^2+y^2+16x+12y+c=0$

centre $(-8,-6)$

Eq of tangent to parabola at $(1,7)$ is

$T=0$

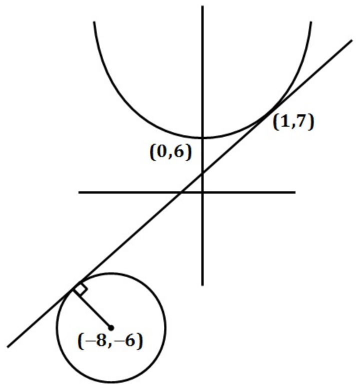
$\frac{y+7}{2}=x\cdot 1+6$

$\Rightarrow y=2x+5$

$\pi=\sqrt{100-c}$

$\sqrt{100-6}=\frac{|-16+6+5|}{\sqrt{4+1}}$

$100-c=\frac{25}{5} \Rightarrow c=95$



Q59. (B)

$\frac{dy}{dx}+y\cdot\cot x=\frac{4x}{\sin x}, \quad x \in (0,\pi)$

$I.F.=e^{\int \cot x \cdot dx}$

$=e^{\int \frac{\cos x}{\sin x}}dx$

Put $\sin x=t \Rightarrow \cos x\cdot dx=dt$

$I.F.=e^{\int \frac{dt}{t}}=e^{\ln t}=\sin x$

$I.F=\sin x$

$\sin x\cdot y=\int \cancel{\sin x}\frac{4x}{\cancel{\sin x}}dx$

$\sin x\cdot y=\cancel{4}\frac{x^2}{2}+c$

at $x=\frac{\pi}{2}\cdot y=0$

$\Rightarrow 0=2\frac{\pi^2}{4}+c \Rightarrow c=\frac{-\pi^2}{2}$

$\sin x\cdot y=2x^2-\frac{\pi^2}{2}$

at $x=\frac{\pi}{6}$

$\frac{1}{2}\cdot y=\cancel{2}\cdot\frac{\pi^2}{\cancel{36}}{}_{18}-\frac{\pi^2}{2}$

$y=-\frac{8\pi^2}{9}$

Q60. (A)

Plane passes through line of intersection of first two planes is

$(2x-2y+3z-2)+\lambda(x-y+z+1)=0$

$x(\lambda+2)-y(2+\lambda)+z(\lambda+3)+(\lambda-2)=0$ (1)

is having infinite number of solution with $x+2y-z-3=0$ and $3x-y+2z-1=0$ then

$$\begin{vmatrix} (\lambda+2) & -(\lambda+2) & (\lambda+3) \\ 1 & 2 & -1 \\ 3 & -1 & 1 \end{vmatrix}=0$$

Solving $\lambda=5$

$7x-7y+8z+3=0$

perpendicular distance from $(0,\ 0,\ 0)$ is $\frac{3}{\sqrt{162}}=\frac{1}{3\sqrt{2}}$.

Q61. (D)

In diffraction

$d sin\ 30^\circ=\lambda$

$\lambda=\frac{d}{2}$

Young's fringe width [d' - separation between two slits]

$\beta=\frac{\lambda\times D}{d'}$

$10^{-2}=\frac{d}{2}\times\frac{50\times10^{-2}}{d'}$

$10^{-2}=\frac{10^{-6}\times50\times10^{-2}}{2\times d'}$

$10^{-2}=\frac{10^{-6}\times50\times10^{-2}}{2\times d'}$

$d'=25\mu m$

Q62. (D)

From Rydberg's formula

$\frac{1}{\Lambda}=R\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$ (1)

From $E=\frac{hc}{\Lambda}=\frac{-13.6}{n^2}$

$\Rightarrow\frac{1}{\Lambda}=\frac{c}{n^2}$ (2)

From (1) & (2)

$\frac{1}{\Lambda}=c\left[\frac{1}{\lambda_g}-\frac{1}{\lambda_n}\right]$

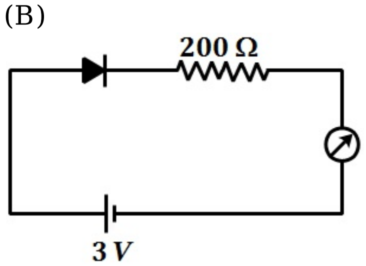
$\Rightarrow\Lambda=\frac{c\lambda_n\lambda_g(\lambda_n+\lambda_g)}{\lambda_n^2-\lambda_g^2}$

$=\frac{c\lambda_n\lambda_g(\lambda_n+\lambda_g)}{\lambda_n^2}$ $[\lambda_n>>\lambda_g]$

$=\frac{c\lambda_n^2\lambda_g}{\lambda_n^2}+\frac{c\lambda_n\lambda_g^2}{\lambda_n^2}$

$\Lambda\sim A+\frac{B}{\lambda_n^2}$

Q63. (B)



Silicon diode is in forward bias.

Hence, across diode potential barrier

$\Delta V=0.7\,V$

$.:I=\frac{V-\Delta V}{R}=\frac{3-0.7}{200}$

$=\frac{2.3}{200}=11.5\ mA$

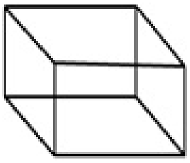
Q64. (B)

$\frac{\Delta \rho}{\rho} = \frac{\Delta m}{m} + \frac{\Delta V}{V}$

$= \frac{\Delta m}{m} + \frac{3\Delta l}{l}$

$1.5\% + 3 \times 1\%$

$= 4.5\%$



Q65. (A)

Radius of circular path in magnetic field is given by

$r = \frac{\sqrt{2Km}}{qB}$

where K = kinetic energy of particle

m = mass of particle

q = charge on particle

B = magnetic field intensity

r = radius of path

For electron

$r_e = \frac{\sqrt{2Km_e}}{eB}$ ... (i)

For proton,

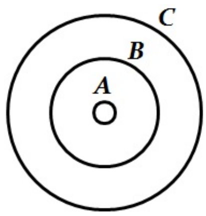
$r_p = \frac{\sqrt{2Km_p}}{eB}$ ...(ii)

For alpha particle

$r_\alpha = \frac{\sqrt{2Km_\alpha}}{q_\alpha B} = \frac{\sqrt{2K4m_p}}{2eB} = \frac{\sqrt{2Km_p}}{eB}$

Since $m_e < m_p$, the order of radius will be $r_e < r_p = r_\alpha$.

Q66. (A)



Potential at B

$=$ (Potential due to A) $+$ (Potential due to B) $+$ (Potential due to C)

Potential due to A:

$\frac{Kq_A}{r_b} = \frac{1}{4\pi\varepsilon_0} \times \frac{4\pi a^2 \times \sigma}{b} = \frac{\sigma}{\varepsilon_0}\frac{a^2}{b}$

Potential due to B:

$\frac{Kq_B}{r_b} = \frac{1}{4\pi\varepsilon_0} \times \frac{4\pi b^2 \times (-\sigma)}{b} = \frac{-\sigma b}{\varepsilon_0}$

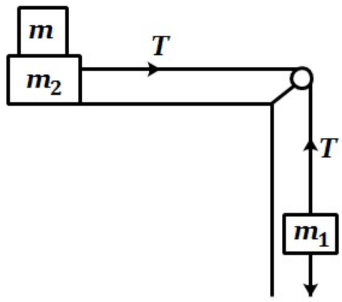
Potential due to C:

$\frac{Kq_C}{r_c} = \frac{1}{4\pi\varepsilon_0} \times \frac{4\pi c^2 \times \sigma}{c} = \frac{\sigma c}{\varepsilon_0}$

Adding all, we get

$V_B = \frac{\sigma}{\varepsilon_0}\frac{a^2}{b} - \frac{\sigma}{\varepsilon_0}b + \frac{\sigma}{\varepsilon_0}c = \frac{\sigma}{\varepsilon_0}\left[\frac{a^2-b^2}{b} + c\right]$

Q67. (A)



FBD

for rest,

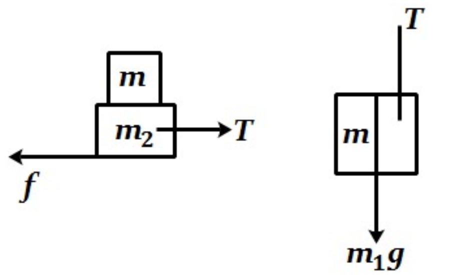
$f = m_1 g$

$m_1 g \le \mu N$ [Since, f is static in nature]

$m_1 \cancel{g} \le \mu(m_2 + m)\cancel{g}$

$\frac{m_1}{\mu} - m_2 \le m$

$\Rightarrow m \ge 23.3\ kg$



Q68. (B)

$U = \frac{-k}{2r^2}$

$F = \frac{-\partial u}{\partial x} = \frac{-k}{r^3}$

Now, since particle is under central action force.

$\Rightarrow \frac{-k}{r^3} = \frac{mv^2}{r}$ [Note $(-)$ sign indicates central acting]

$\Rightarrow \left|\frac{1}{2}mv^2\right| = \frac{k}{2r^2}$

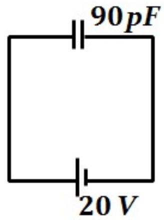
Now, Total Energy

$= u|_a + k|_a$

$= \frac{-k}{2a^2} + \frac{k}{2a^2}$

$= 0$

Q69. (D)



$\sigma_i = \sigma\left(1 - \frac{1}{k}\right)$

$\Rightarrow \sigma_i = \sigma\left(1 - \frac{1}{\left(\frac{5}{3}\right)}\right)$

$\Rightarrow \sigma_i = \frac{2}{5}\sigma$

$\Rightarrow Q_i = \frac{2}{5}(A) \times (E\varepsilon_0) \times \left(\frac{d}{d}\right)$

$= \frac{2}{5} \times C \times V$

$= \left(\frac{2}{5} \times 90 \times 20 \times \frac{5}{3}\right) pC$

$= 1200\ pC$

$= 1.2\ n\ C$

Q70. (A)

For silver atom,

no. of atoms in 1 mole $= N_A$

We know that

$\sqrt{\frac{k}{\mu}} = 2\pi f$ [where μ is reduced mass]

$\Rightarrow \frac{k}{\mu} = 4\pi^2 f^2$

$\Rightarrow k = 4\pi^2 f^2 \mu$

$= 4\pi^2 f^2 \frac{m}{N_A}$

$= 4\pi^2 \times (10^{12})^2 \times \frac{0.108}{6.02 \times 10^{23}}$

$= 7.1\ N/m$

Q71. (D)

A neutron and a deuterium are of comparable masses, and under elastic collision conditions, analytically we can say, that the deuterium will gain a final speed comparable to neutron's initial speed.

This in energy terms for the neutron can be stated as a high amount of loss as it almost transfers all of its energy to deuterium.

However, with carbon nucleus (24 times the mass of neutron), the neutron will bounce back will almost the same speed, resulting in very small loss or small transference of its kinetic energy.

Option D only satisfies the above logic.

Q72. (B)

$m_1 = IA$

$M_2 = 2m = IA'$

$A_2 = 2A_1$

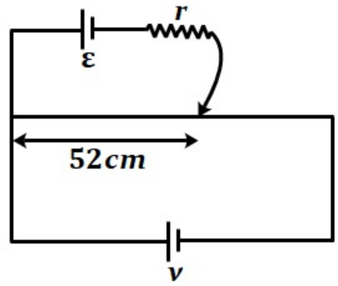
$\therefore r_2 = \sqrt{2}r_1$

$B_1 = \frac{\mu_0 I}{2r_1}$

$B_2 = \frac{\mu_0 I}{2(\sqrt{2}r_1)}$

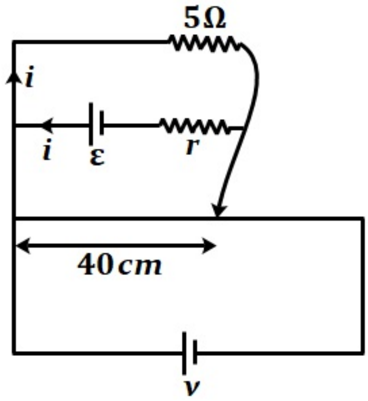
$\frac{B_1}{B_2} = \sqrt{2}$

Q73. (A)



$\therefore \varepsilon = 0.52\,V$

Now



$0.4\,V = 5i$

$i = \frac{0.4\,V}{5}$

$\varepsilon - ir = 0.4\,V$

$0.52\,V - \frac{0.4\,V}{5}r = 0.4\,V$

$0.12\,V = \frac{0.4\,V}{5}r$

$\frac{12}{40} \times 5 = r$

$r = 1.5\,\Omega$.

Q74. (B)

$f = 10\,GHz$

Available $f = 10\% f = \frac{10}{100} \times 10\,GHz$

$= 1\,GHz$

Channels $= \frac{\text{Available } f}{\text{Bandwidth}} = \frac{1\,GHz}{5} = 2 \times 10^5$.

Q75. (B)

A & B pass axis are parallel.

$\frac{I}{2}\cos^2\theta = I'$ (1)

Let I' be the intensity passing through 'C'

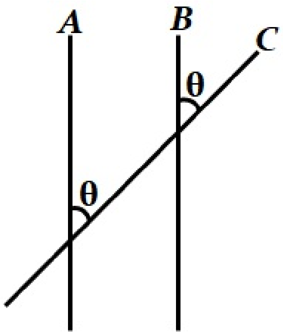
$\therefore I'\cos^2\theta = \frac{I}{8}$ (2)

$\frac{I}{2}\cos^2\theta = \frac{I}{8\cos^2\theta}$

$\cos^4\theta = \frac{1}{4}$

$\cos\theta = \frac{1}{\sqrt{2}}$

$\theta = 45^\circ$.



Q76. (B)

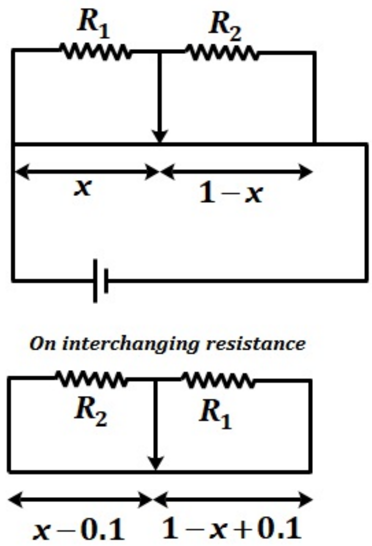
$\frac{R_1}{R_2} = \frac{x}{1-x}$

$\frac{R_2}{R_1} = \frac{x-0.1}{1.1-x}$

& $R_1 + R_2 = 1000\,\Omega$

On solving the three equations we get

$R_1 = 550\,\Omega$ & $R_2 = 450\,\Omega$.



Q77. (D)

Using Superposition Principle

$I_{\text{entire disc}} = I_{\text{smaller part}} + I_{\text{remaining part}}$

$\frac{9MR^2}{2} = \left[\frac{M\left(\frac{r}{3}\right)^2}{2} + M\left(\frac{2R}{3}\right)^2\right] + I_{\text{remaining part}}$

$\frac{9MR^2}{2} = I_{\text{remaining part}} + \frac{9MR^2}{18}$ [Mass of smaller position will be $\frac{9M}{9}$, since area of smaller disc is $\frac{1}{9}$ times of entire disc.]

$\frac{9MR^2}{2} = I_{\text{remaining part}} + \frac{MR^2}{2}$

$I = 4\,MR^2$

Q78. (A)

$mv_0 = mv_1 + mv_2$

$v_0 = v_1 + v_2$

And $KE_f = \frac{150}{100}KE_i$

$\frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 = \frac{3}{2}\left(\frac{1}{2}mv_0^2\right)$

Putting $v_1 = v_0 - v_2$

$v_2 = \frac{v_0}{2} - \frac{v_0}{\sqrt{2}}$ & $v_1 = \frac{v_0}{2} + \frac{v_0}{\sqrt{2}}$

$\therefore$ Relative velocity $= \frac{v_0}{\sqrt{2}} - \left(\frac{-v_0}{\sqrt{2}}\right) = \sqrt{2}\,v_0$.

Q79. (B)

Let air be referred to as medium 1 and the medium as medium 2.

$k = \frac{2\pi\nu}{c} = \frac{2\pi}{\lambda}$

where $k_2 = 2k = \frac{2\pi}{\lambda_2}$

$\therefore \lambda_2 = \frac{\lambda}{2} \Rightarrow v_2 = \frac{c}{2}$

$\therefore n_2$ (refractive index of medium 2)

$n_2 = \frac{c}{v_2} = 2$

$\frac{n_2}{n_1} = \sqrt{\frac{\mu_2\varepsilon_2}{\mu_1\varepsilon_1}}$

$2 = \sqrt{\frac{\varepsilon_2}{\varepsilon_1}}\left(\frac{\mu_2}{\mu_1} = 1, \because \text{medium 2 is non-magnetic}\right)$

$\therefore \frac{\varepsilon_1}{\varepsilon_2} = \frac{1}{4}$.

Q80. (D)

Quality factor $= \frac{\omega_o L}{R}$

Q81. (A)

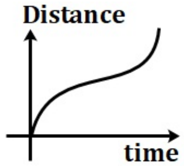
$x=-(t-a)^{2}+b$

$v=-2(t-a)$

$\frac{dv}{du}=\frac{\frac{dv}{dt}}{\frac{du}{dt}}$

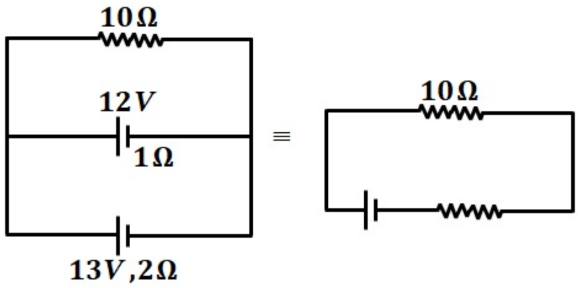
$\frac{-2}{-2(t-a)}=\frac{1}{t-a}$

A) is incorrect because from $t=0$ to $t=a$, position and distance should be same.



$\rightarrow$ this should be option A).

Q82. (A)



$\varepsilon_{\text{eq}}=\frac{\frac{\varepsilon_1}{r_1}+\frac{\varepsilon_2}{r_2}}{\frac{1}{r_1}+\frac{1}{r_2}}=\frac{\frac{12}{1}+\frac{13}{2}}{\frac{1}{1}+\frac{1}{2}}$

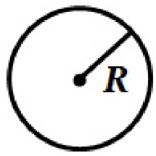
$=\frac{37}{3}V$

$r_{\text{eq}}=\frac{1}{\frac{1}{r_1}+\frac{1}{r_2}}=\frac{2}{3}\Omega$

$V=\frac{10}{10+\frac{2}{3}}\times\frac{37}{3}=\frac{10\times 37}{3\Omega}$

$=11.5625$ volt.

Q83. (B)



$F\propto\frac{1}{R^{n}}$

$\frac{mv^{2}}{R}\propto\frac{1}{R^{n}}$

$\Rightarrow v\propto R^{\frac{(1-n)}{2}}$

$T=\frac{2\pi R}{v}\propto\frac{R}{R^{\frac{(1-n)}{2}}}=R^{\left(\frac{n+1}{2}\right)}$.

Q84. (C)

$h\nu=E_{o}\left[\frac{1}{n_1^{2}}-\frac{1}{n_2^{2}}\right]$

for Lyman series, series limit $n_2=\infty, n_1=1$

$\Rightarrow \nu_{\text{Lyman}}=\frac{c}{\lambda}=RC\left[1-\frac{1}{\infty}\right]=\nu_L$

for Pfund series, series limit $n_2=\infty, n_1=5$

$\Rightarrow \nu_{\text{Pfund}}=RC\left[\frac{1}{5^{2}}-\frac{1}{\infty}\right]=\frac{1}{25}RC=\frac{\nu_L}{25}$.

Q85. (A)

Average power $=V_{\text{RMS}}I_{\text{RMS}}\cos\phi$

$=\frac{100}{\sqrt{2}}\times\frac{20}{\sqrt{2}}\times\cos\frac{\pi}{4}$

$=\frac{1000}{\sqrt{2}}$ Watts.

Wattless current $=I_{ms}\sin\theta$

$=\frac{I_o}{\sqrt{2}}\sin 45^{\circ}$

$=\frac{20}{\sqrt{2}}\sin 45^{\circ}$

$=10\ Amp$

Q86. (B)

$n = 2,\ V,\ T = 27^{\circ}C = 300\ K$

$n_f = 2,\ V_f = 2V,\ T_f = ?$

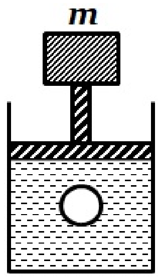
$T_1 V_1^{\gamma-1} = T_2 V_2^{\gamma-1}$

$\Rightarrow T_2 = \frac{300 \times V^{\gamma-1}}{(2V)^{\gamma-1}} = \frac{300}{2^{\frac{5}{3}-1}} = \frac{300}{2^{\left(\frac{2}{3}\right)}}$

$= 189\ K$

$\Delta U = nC_V\,\Delta T = 2 \times \frac{3R}{2} \times (189 - 300) = -2.7\ kJ$.

Q87. (B)

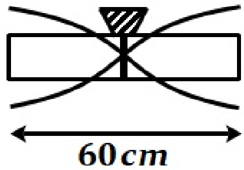


$\Delta P = \frac{mg}{a}$

$\left|\frac{\Delta V}{V}\right| = \left|\frac{-\Delta P}{B}\right| = \frac{mg}{aB}$

$\frac{\Delta V}{V} = 3\frac{\Delta r}{r} \Rightarrow \frac{\Delta r}{r} = \frac{mg}{3\ aB} = \frac{mg}{3\ aK}$.

Q88. (D)



$\rho = 2.7 \times 10^3\ kg/m^3$

$Y = 9.27 \times 10^{10}\ Pa$

$\frac{\lambda}{2} = L$

$\frac{\lambda}{2} = 60\ cm \Rightarrow \lambda = 120\ cm$

Velocity $V = \sqrt{\frac{Y}{\rho}} = \sqrt{\frac{9.27 \times 10^{10}}{2.7 \times 10^3}} = 5.86 \times 10^3\ m/s$

frequency $f = \frac{V}{\lambda} = \frac{5.84 \times 10^3}{1.2} = 4882\ Hz$.

Q89. (D)

$\text{Pressure} = \frac{\text{Force}}{\text{Area}} = \frac{\left(\frac{\Delta P}{\Delta f}\right)}{\text{Area}} = \frac{\left(\frac{2\ mv\ \cos\theta}{1\ \sec}\right)}{\text{Area}}$

$= \frac{2 \times 3.32 \times 10^{-27} \times 10^3 \times 10^{23} \times \cos 45^{\circ}}{2 \times 10^{-4}}$

$= 2.35 \times 10^3\ N/m^2$.

Q90. (C)

$I = I_{\text{CM}} + MR^2$

$I_{\text{CM}} = I_0 = \frac{MR^2}{2} + \left[\frac{MR^2}{2} + M\left(4R^2\right)\right] \times 6 = \frac{55}{2}MR^2$

$I_P = \frac{55}{2}MR^2 + (7M)\left((3R)^2\right) = \frac{181\ MR^2}{2}$